DOI: https://doi.org/10.36910/6775-2524-0560-2026-64-36
УДК 621.391.7:004.8:519.876.5
Gariachiy Maksim, PhD student
https://orcid.org/0000-0001-5177-6958
Shcherbinin Serhii, PhD in Engineering
https://orcid.org/0000-0003-0966-2476
Scientific Center of the Air Force Ivan Kozhedub Kharkiv National University of Air Forces, Kharkiv, Ukraine

THE METHOD FOR STOCHASTIC SIGNAL FORMATION USING THE KARHUNEN–LOÈVE TRANSFORM

Gariachiy M., Shcherbinin S. The method for stochastic signal formation using the Karhunen–Loève transform. The article addresses the problem of improving the spectral efficiency of stochastic signals in information and control systems operating under limited radio-frequency resources, nonlinear distortions, and interference. It is shown that conventional signal generation and processing techniques do not provide a stable reduction in spectral redundancy for a broad class of stochastic processes. Moreover, the use of nonlinear models without accounting for the statistical structure of the input signal may lead to deterioration of its spectral characteristics and an increase in model complexity. An improved stochastic signal generation algorithm based on the combined use of the Karhunen–Loève transform and a nonlinear Volterra model is proposed. The key feature of the approach is the implementation of nonlinear processing in the principal-component domain. This provides decorrelation of signal components, energy concentration, and better matching of the Volterra kernel structure to the statistical properties of the processed signal. As a result, the parametric complexity of the nonlinear model can be reduced while enabling controlled formation of the output spectral characteristics. Numerical simulation results demonstrate that, for structured stochastic signals, the proposed algorithm reduces the effective spectral bandwidth and improves spectral efficiency. For noise-like signals, however, nonlinear processing without prior statistical decorrelation is shown to be less effective. The obtained results confirm the feasibility of combining orthogonal statistical decomposition with nonlinear modelling for generating stochastic signals with specified spectral properties. The proposed approach can be applied in control, communication, and radar systems requiring improved spectral efficiency and robustness to interference.
Keywords: stochastic signals, spectral efficiency, Karhunen–Loève transform, Volterra series, nonlinear modeling.

Гарячий М. В., Щербінін С. О. Метод формування стохастичних сигналів із використанням перетворення Кархунена-Лоєва. У статті вирішено задачу підвищення спектральної ефективності стохастичних сигналів в інформаційних системах управління в умовах обмеженого радіочастотного ресурсу та наявності нелінійних спотворень і завад. Доказано що класичні методи формування та оброблення сигналів не забезпечують стабільного зменшення спектральної надмірності для широкого класу стохастичних процесів, а застосування нелінійних моделей без урахування статистичної структури сигналу може призводити до погіршення спектральних характеристик. Запропоновано вдосконалений алгоритм формування стохастичних сигналів, що ґрунтується на комбінованому використанні перетворення Кархунена–Лоєва та нелінійної моделі Вольтерра. Ключовою особливістю підходу є перенесення нелінійної обробки у простір головних компонент, що забезпечує декореляцію складових сигналу, концентрацію енергії та узгодження структури ядра з його статистичними властивостями. Це дозволяє зменшити параметричну складність моделі та забезпечити кероване формування спектральних характеристик. Результати чисельного моделювання підтвердили, що для структурованих стохастичних сигналів запропонований алгоритм забезпечує зменшення ефективної ширини спектра та підвищення спектральної ефективності, тоді як для шумоподібних сигналів без попередньої статистичної декореляції ефективність нелінійної обробки знижується. Отримані результати свідчать про доцільність поєднання ортогонального статистичного розкладу та нелінійного моделювання для формування стохастичних сигналів із заданими спектральними характеристиками. Запропонований підхід може бути використаний у системах управління, зв'язку та радіолокації з підвищеними вимогами до спектральної ефективності та завадостійкості.
Ключові слова: стохастичні сигнали, спектральна ефективність, перетворення Кархунена–Лоєва, ряди Вольтера, нелінійне моделювання, декореляція, інформаційні системи управління.

Statement of a scientific problem.

Modern information control systems operate under conditions of severe scarcity of radio frequency resources and high levels of both natural and artificial interference. The transmission of stochastic signals in such systems requires high spectral efficiency, robustness, and adaptability to time-varying statistical characteristics of the communication channel. Conventional signal design methods based on linear models or fixed orthogonal transforms exhibit limited performance when dealing with signals characterized by complex nonlinear structures and strong temporal correlations.

Recent studies demonstrate the potential of the Karhunen–Loève Transform (KLT) for compact spectral representation and energy concentration of stochastic processes [1–3]. At the same time, research in adaptive signal processing for radar and communication systems shows that the KLT is an effective tool for noise suppression, component decorrelation, and improvement of detection performance [4, 5].

However, the standalone application of the KLT is limited in problems where nonlinear interactions and memory effects of stochastic processes play a significant role.

On the other hand, Volterra series represent one of the most powerful mathematical tools for modeling nonlinear systems with memory, enabling the description of complex dependencies in stochastic signal time series [6, 7]. However, their practical application remains challenging due to high parametric dimensionality, regularization issues, and the accumulation of statistical errors during kernel estimation.

Nevertheless, the problem of developing a method for the formation and optimization of stochastic signals remains unresolved. Such a method should simultaneously account for nonlinear interactions and memory effects of the process, ensure minimization of spectral redundancy and efficient energy representation, possess adaptability to time-varying statistical characteristics of the communication channel, and improve interference immunity as well as the probability of correct detection under complex electromagnetic environments.

Therefore, the search for an integrated approach that combines the capabilities of Volterra series with the optimization properties of the Karhunen–Loève Transform is a relevant and important scientific task.

Research analysis.

Based on the analysis of scientific publications devoted to orthogonal expansions of stochastic processes, it has been established that the Karhunen–Loève Transform (KLT) is one of the most theoretically grounded tools for representing random signals. In the fundamental works of Loève and Kosambi, the Kosambi–Karhunen–Loève theorem was formulated, which defines the optimal representation of a stochastic process in the basis of eigenfunctions of the covariance operator and ensures the minimum mean-square approximation error for a limited number of components [1, 8]. The further development of these ideas is reflected in principal component analysis (PCA), which represents a discrete analogue of the KLT, as confirmed by generalized results of survey studies [2].

At the same time, an analysis of contemporary studies shows that PCA and KLT are predominantly employed as tools for statistical analysis, data compression, and dimensionality reduction. In particular, in problems of stochastic modeling and reliability analysis, the KLT is mainly used for random field reduction and improving the computational efficiency of numerical methods [9–13]. However, such approaches are primarily focused on analysis and numerical modeling rather than on the synthesis of stochastic signals with prescribed spectral characteristics under conditions of limited frequency resources.

A separate line of research is associated with the application of the KLT to enhance interference immunity and improve signal detection performance. It has been shown that adapting the basis to the statistical properties of the process provides superior performance compared to classical spectral methods, particularly in tasks such as weak signal detection, signal-to-noise ratio improvement, and suppression of nonstationary interference [4, 5, 14-16]. At the same time, in these studies, the KLT is predominantly applied at the stage of received signal processing, which limits the ability to influence the spectral characteristics of signals at the stage of their synthesis.

The literature also highlights that the choice of an orthogonal basis significantly influences the spectral localization and redundancy of signals. However, the problem of purposeful synthesis of stochastic signals with optimized spectral characteristics is scarcely addressed within these approaches [17].

In parallel, nonlinear models for signal generation and processing are widely studied, particularly Volterra series and their modifications, which enable the consideration of system nonlinearity and memory effects [6, 7, 18]. However, it is emphasized that as the order of nonlinearity increases, the parametric complexity of such models grows rapidly, while the lack of consistency with the statistical structure of signals may lead to degradation of their spectral characteristics.

Local studies have primarily focused on the application of the KLT in signal compression, recognition, and reconstruction, particularly through the optimization of spectral component sets based on the probability of correct recognition [19–21]. Meanwhile, issues related to improving spectral efficiency in communication channels remain of secondary importance.

It can therefore be concluded that the Karhunen–Loève Transform is an effective tool for decorrelation and energy concentration of stochastic signals [1, 2, 9, 12, 13]. However, its potential as a means for the synthesis of stochastic signals with controlled spectral characteristics and enhanced interference immunity remains insufficiently exploited. This indicates the existence of a scientific gap related to the development of methods for stochastic signal synthesis based on the KLT, aimed at improving spectral efficiency under conditions of limited radio frequency resources.

The purpose of the work. The aim of this work is to develop and improve methods for the synthesis of stochastic signals based on the Karhunen–Loève Transform in order to enhance spectral efficiency in electronic communication systems.

Presentation of the main material and substantiation of the obtained research results.

In modern information management systems, the problem of efficient utilization of limited radio frequency resources is becoming increasingly important. Stochastic signals, despite their inherent advantages in terms of interference immunity and low probability of detection, are characterized by a non-uniform energy distribution in the spectral domain, which results in increased spectral redundancy and an effective widening of the occupied bandwidth. This limits their applicability in systems where spectral efficiency is a critical performance metric.

Conventional methods of signal generation and processing, including time–frequency transforms and linear filtering techniques, do not provide a sufficient level of adaptation to the statistical properties of stochastic processes. The use of nonlinear models, particularly Volterra series, enables the consideration of memory effects and nonlinear interactions between signal samples. However, without prior alignment with the statistical structure of the signal, such approaches may lead to uncontrolled spectral expansion and increased parametric complexity.

Therefore, it is advisable to employ methods capable of reducing the internal statistical redundancy of stochastic signals already at the stage of their synthesis. The Karhunen–Loève Transform provides optimal decorrelation of the components of a stochastic process and concentrates energy within a limited number of principal components, thereby enabling purposeful control of the signal's spectral characteristics.

The combination of such a statistically consistent representation with nonlinear processing enables the synthesis of signals with improved spectral properties without significant loss of information content [6].

In view of the above, this paper proposes an algorithm for the synthesis of stochastic signals based on the combined use of the Karhunen–Loève Transform and nonlinear modeling. This approach enables a reduction in spectral redundancy, ensures consistency of nonlinear processing with the statistical structure of the signal, and creates conditions for improving spectral efficiency in systems with limited frequency resources.

The analysis of the results and limitations of existing approaches shows that the efficiency of stochastic signal synthesis based on Volterra series significantly depends on the consistency of the kernel structure with the internal statistical properties of the signal. As demonstrated in previous studies, fixed or weakly adaptive models improve spectral characteristics primarily for signals with stationary or structured correlation properties, whereas for broadband stochastic realizations they may lead to the opposite effect – namely, an increase in spectral redundancy and an expansion of the effective bandwidth.

This necessitates a transition from isolated nonlinear processing procedures to a holistic algorithmic framework that integrates preliminary statistical signal conditioning, adaptive kernel structure formation, and spectral efficiency control using feedback mechanisms. Such a step-by-step organization of the process is implemented in the proposed stochastic signal synthesis algorithm.

To ensure a stable increase in spectral efficiency, an adaptive feedback criterion based on the evaluation of spectral efficiency variation is incorporated into the algorithm. The Karhunen–Loève Transform is applied only if the energy criterion (1) is satisfied, after which the result of nonlinear processing is evaluated using the spectral efficiency metric ΔSE. In the case of a negative or insufficient increase in spectral efficiency, the parameters of the nonlinear model are adaptively adjusted, which allows avoidance of spectral expansion regimes and ensures that the algorithm operates within the region of effective performance.

Formally, the decision criterion for the application of the KLT is defined as (1):

$$\begin{cases} \sum_{i=1}^{k} \lambda_i \geq \eta \sum_{i=1}^{L} \lambda_i \\ \Delta SE \geq \varepsilon SE \end{cases}_{min} \qquad (1)$$

where λ_i – the eigenvalues of the signal covariance matrix obtained as a result of the Karhunen–Loève Transform;

L – the full dimension of the feature space;

k – the number of retained principal components;
η_{min}– the minimum acceptable fraction of cumulative energy retained in the first k components (typically 0.9);
ΔSE– the relative change in spectral efficiency after processing;
εSE– the threshold value of spectral efficiency improvement considered sufficient.

A block diagram of the algorithm for optimizing the spectral characteristics of stochastic signals based on the combination of Volterra series and the Karhunen–Loève Transform is presented in Fig. 1.

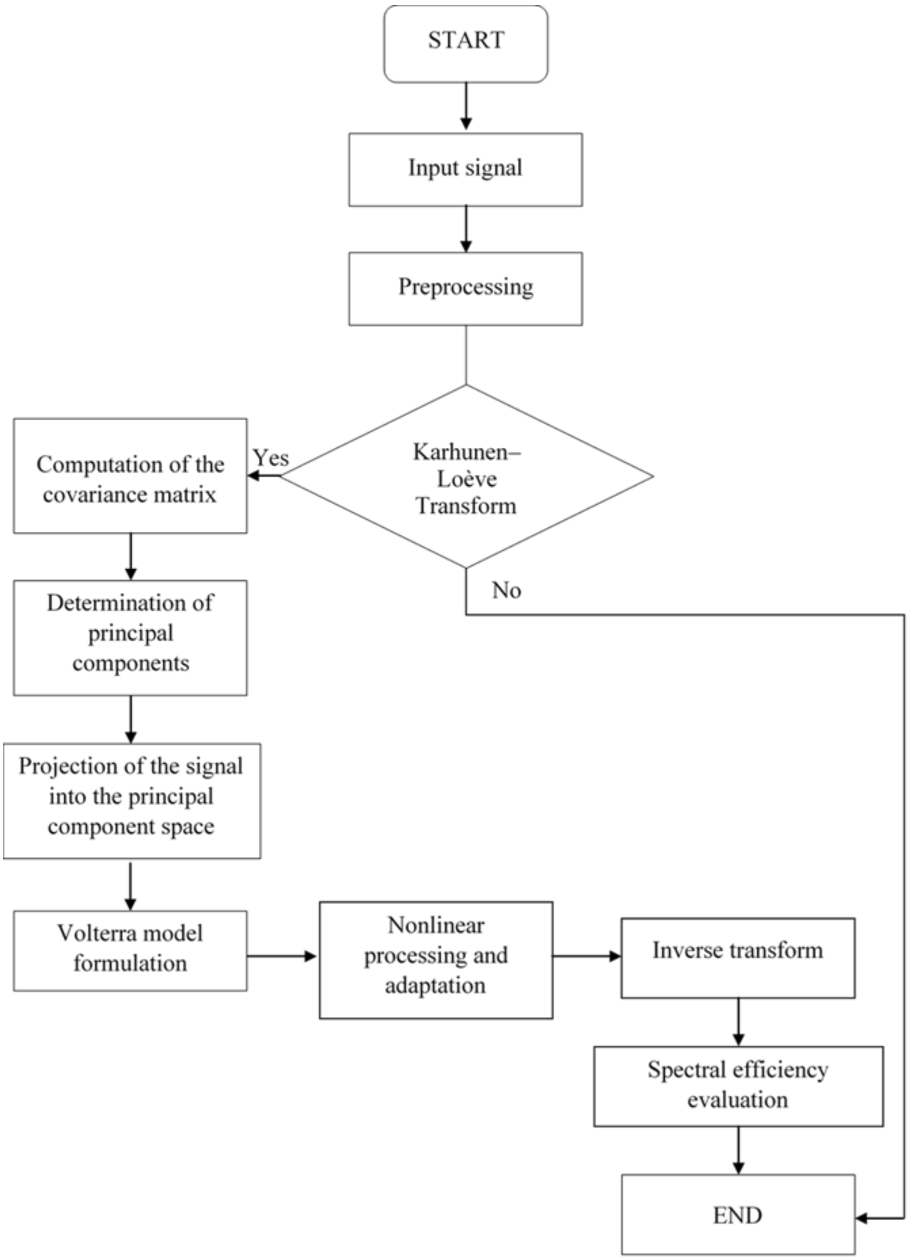

Figure 1 – Block diagram of the algorithm for optimizing the spectral characteristics of stochastic signals based on the combination of Volterra series and the Karhunen–Loève Transform

As shown in Fig. 1, the algorithm implements a combined approach in which preliminary statistical signal processing is complemented by optimization in the space of principal components. This sequence makes it possible to reduce spectral redundancy and improve spectral efficiency through the use of the Karhunen–Loève Transform and a nonlinear Volterra model while maintaining reduced computational complexity. The proposed algorithm represents a further development of the approach presented in previous studies (Fig. 2) [22].

In contrast to the baseline scheme, the improved algorithm implements step-by-step statistical signal conditioning, adaptive formation of the kernel structure, and spectral efficiency control using a feedback mechanism. The key enhancement lies in aligning the parameters of the Volterra kernel with the internal statistical properties of the signal, which makes it possible to prevent uncontrolled spectral expansion in broadband stochastic realizations while preserving the beneficial spectral compression effect for structured signals. This approach ensures a stable improvement in spectral efficiency under conditions of limited frequency resources and makes the algorithm suitable for practical application in information control systems with nonlinear distortions and memory effects.

The main stages of the algorithm for optimizing the spectral characteristics of stochastic signals based on the combination of Volterra series and the Karhunen–Loève Transform are as follows:

1. Representation of the input stochastic signal.
2. Preprocessing of the signal, including normalization and removal of the DC component.
3. Application of the Karhunen–Loève Transform.
4. Extraction of principal components and projection of the signal onto the principal component space.

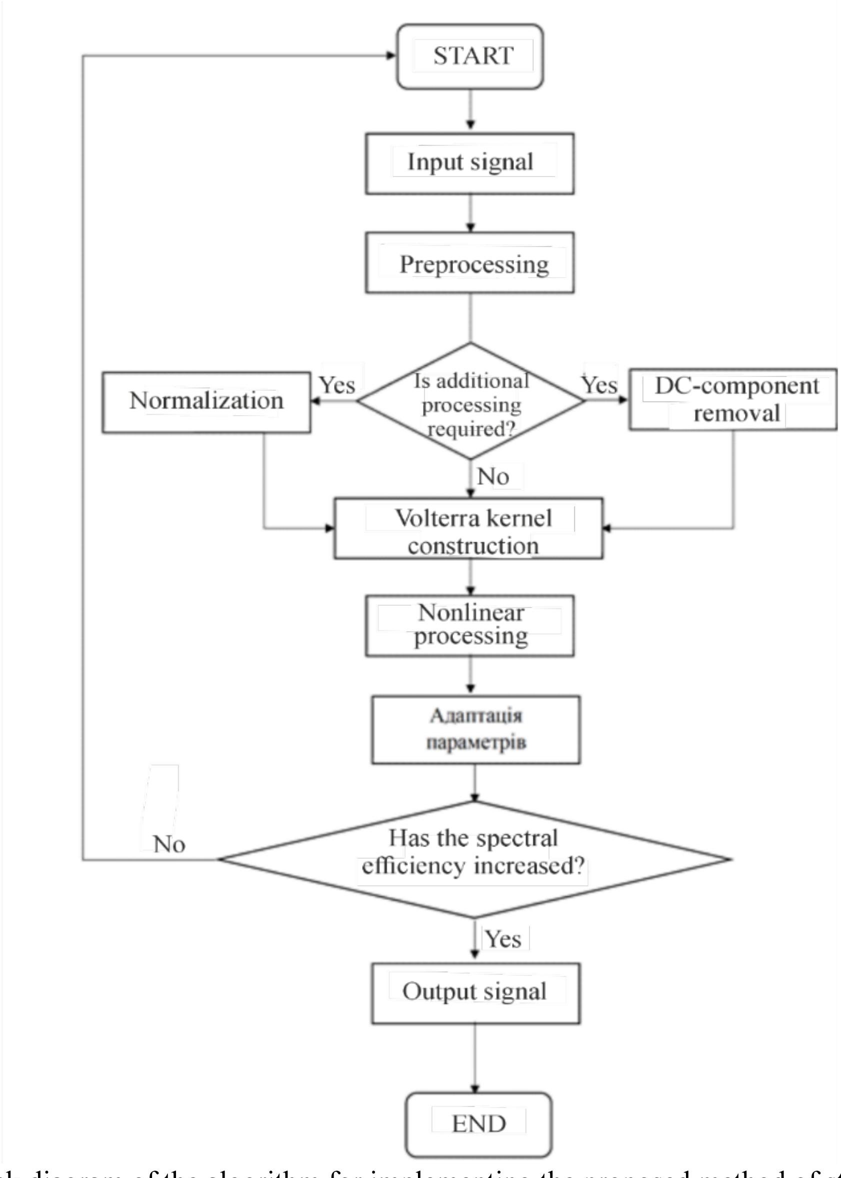

Figure 2 – Block diagram of the algorithm for implementing the proposed method of stochastic signal synthesis and nonlinear processing based on Volterra series

5. Formation of a nonlinear model based on Volterra series.
6. Nonlinear signal processing and adaptive adjustment of model parameters.

7. Inverse transformation from the principal component space to the time domain.
8. Evaluation of the spectral efficiency of the synthesized signal.

At the first stage of the algorithm, the input stochastic signal is preprocessed. The purpose of this stage is to ensure the correctness and efficiency of signal optimization through normalization, i.e., adjusting the signal amplitude to achieve the desired energy characteristics, and centering, i.e., ensuring a zero mean of the signal. Centering is performed to obtain a symmetric covariance matrix and to eliminate parasitic components.

During the preprocessing stage, normalization is carried out with respect to both amplitude and total energy of the input signal. While amplitude normalization limits the range of signal values and prevents excessive computational complexity, energy normalization ensures consistency of signal energy characteristics and enables stable extraction of principal components.

Normalization with respect to total energy (2) is performed such that the normalized signal has unit total energy.

$$E = \sum_{n=1}^{N} |x(n)^2|, \tag{2}$$

where E – total energy of the signal;
$x(n)$ – input signal;
N – number of samples.

The normalized signal is defined as (3):

$$x_{norm} = \frac{x(n)}{\sqrt{E}}. \tag{3}$$

Normalization with respect to total energy eliminates the influence of signal energy on the decomposition results. In the proposed algorithm, a combination of amplitude and energy normalization is employed, as defined in (4):

$$x_{norm}[n] = \frac{x[n]}{\left|x[n]\right|_{\max}} \cdot \sqrt{\frac{E_0}{\sum_{n=1}^{N}\left(\frac{x[n]}{\left|x[n]\right|_{\max}}\right)^2}} \tag{4}$$

where $x_{norm}[n]$ – normalized signal at the n-th sample;
$x[n]$ – input signal at the n-th sample;
E_0 – normalized signal energy value;
N – number of samples.

According to (3), the signal is sequentially normalized through amplitude limiting and energy matching. First, the signal is scaled such that the maximum absolute amplitude becomes equal to unity, which prevents the occurrence of excessive peak values in the processed signal. Subsequently, the total signal energy is adjusted to match its normalized value. Satisfying this condition ensures uniform energy conditions for different realizations of the stochastic signal when analyzing processing results and spectral efficiency.

Fig. 3 illustrates an example of combined normalization of filtered white noise.

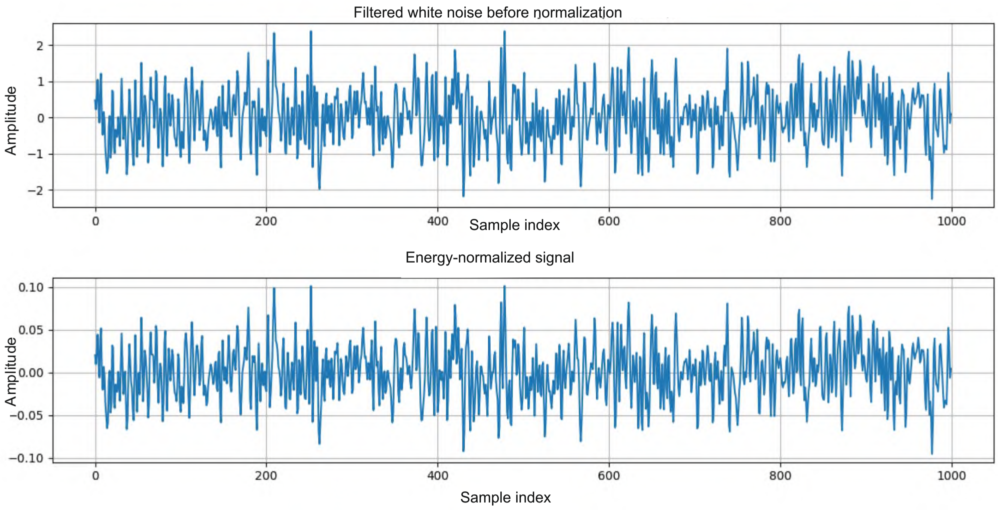

Figure 3 – An example of combined normalization of filtered white noise

As shown in Fig. 3, after applying combined normalization, the amplitude of the filtered white noise is reduced in such a way as to ensure uniformity of both the maximum amplitude and the total energy, while the waveform remains unchanged.

Thus, preliminary energy normalization is an essential stage in the preparation of stochastic signals for spectral optimization and decorrelation tasks aimed at improving data processing efficiency in information systems.

Based on the normalization results, a decision is made to perform signal centering (Fig. 4). The purpose of centering is to eliminate the mean value of the signal samples. This requirement arises from the fact that the signal energy should be associated with its fluctuating component rather than with its constant component. If the mean value of the stochastic signal is nonzero, this may increase the risk of the appearance of a zero-frequency component in the signal spectrum, which leads to incorrect decorrelation of signal components.

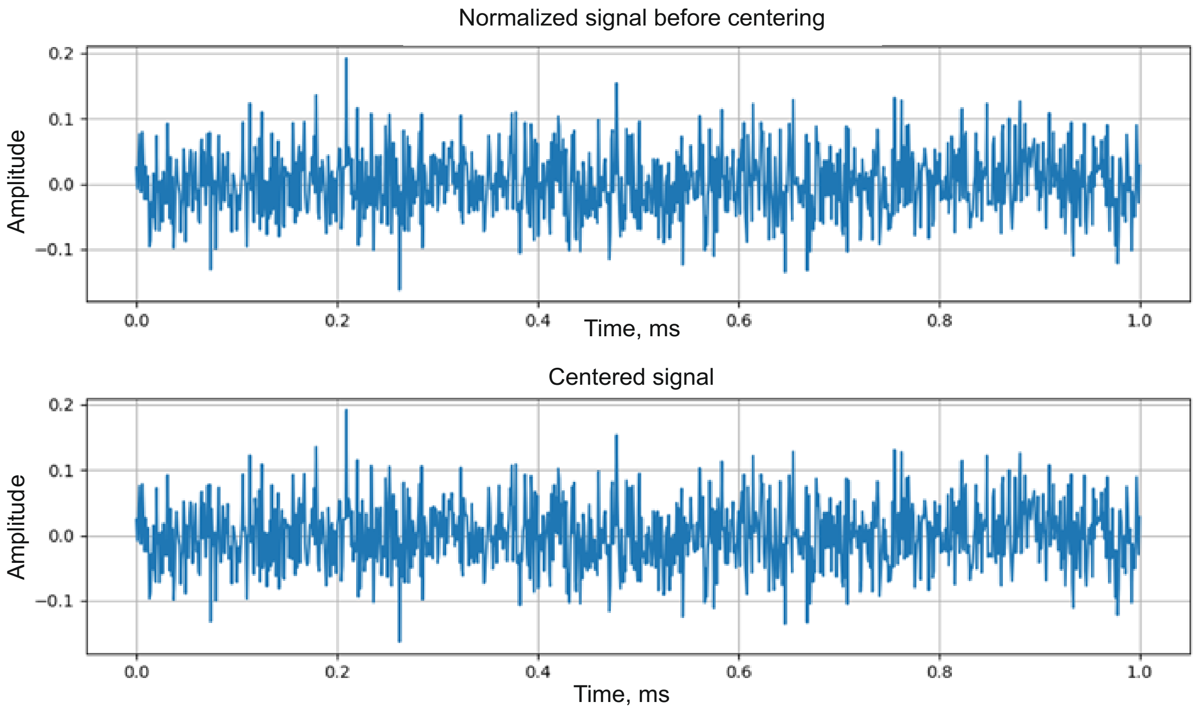

Figure 4 – An example of centering normalized filtered white noise

As shown in Fig. 4, after the centering operation, the mean level of the signal is shifted to zero. Although the waveform and stochastic nature of the signal fluctuations are preserved, its constant component (which could lead to the appearance of energy at zero frequency) is eliminated. This ensures the removal of parasitic DC offsets.

Following the preprocessing steps, which include normalization of the stochastic signal in terms of energy and amplitude, as well as centering, the signal is processed using the Karhunen–Loève Transform.

The Karhunen–Loève Transform involves the construction of the covariance matrix, computation of eigenvalues and eigenvectors, and projection of the signal onto the principal component space. A detailed mathematical description of this procedure is provided in the previous sections.

Prior to applying the Karhunen–Loève Transform, a feature matrix is formed (Fig. 5) for subsequent processing.

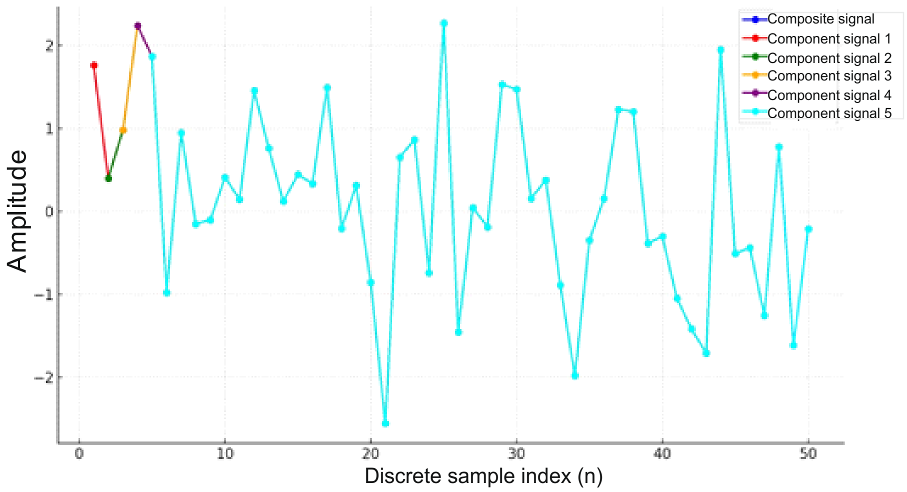

Figure 5 – Feature matrix construction based on the signal

Fig. 5 illustrates the process of feature matrix construction based on the signal. The signal is divided into short segments of fixed length (features), which are sequentially extracted along the signal samples. Each segment represents an individual feature vector that is subsequently used to construct the covariance matrix in the Karhunen–Loève Transform. The extracted features (shown in different colors) reflect the local structure of the signal, ensuring the adaptability of the method to the statistical properties of the process.

The normalized and centered signal $x[n]$ is represented as a set of sequences (segments), the total number of which is L, each containing N samples. Segmenting the signal allows obtaining a statistically reliable estimate of the covariance matrix based on multiple realizations of the process, provides a locally averaged assessment of correlations between samples, and prepares the data for effective extraction of principal components using the Karhunen–Loève Transform.

As a result, a matrix X of size $N \times L$ is formed, where N is the window length (number of samples in each segment), and L is the number of segments obtained by sliding the window along the signal, (5):

$$X = \begin{bmatrix} x_1[1] & x_2[1] & . & . & . & x_L[1] \\ x_1[2] & x_2[2] & . & . & . & x_L[2] \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ x_1[N] & x_2[N] & . & . & . & x_L[N] \end{bmatrix}, \tag{5}$$

where $x_l[n]$ – the n-th sample in the l-th segment.

Based on the constructed feature matrix, the covariance matrix of the signal is computed as (6):

$$K = \frac{1}{L} X X^T \tag{6}$$

where $K \in \mathbb{R}^{M \times M}$ – the feature covariance matrix;

X^T – the transposed feature matrix.

Based on the constructed covariance matrix, the eigenvalues and eigenvectors are computed (Fig. 6).

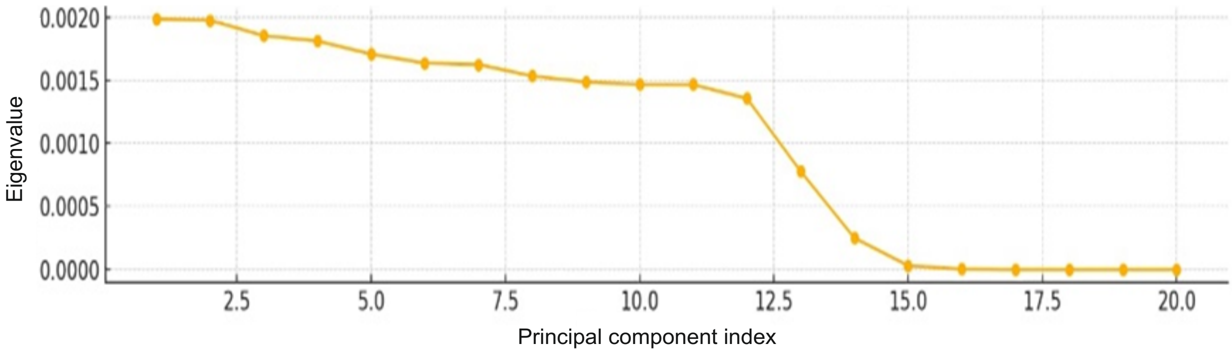

Figure 6 – Eigenvalue spectrum of the covariance matrix

As shown in Fig. 6, the eigenvalue spectrum of the covariance matrix exhibits a pronounced decaying behavior: the first few eigenvalues significantly exceed the subsequent ones in magnitude. This indicates that the majority of the signal energy is concentrated in a small number of principal components, while the remaining components contribute only marginally to the total energy of the process.

Such behavior of the spectrum is typical for stochastic signals with inherent redundancy and justifies the elimination of low-informative components in order to reduce dimensionality and optimize the spectral characteristics of the signal.

Fig. 7 shows the eigenvalue distribution of the covariance matrix, illustrating the number of components required to ensure the reconstruction of the total signal energy. As can be seen from the figure, the first 10 principal components allow almost complete recovery of the signal energy. Further increase in the number of components has only a negligible effect on energy reconstruction, indicating their low informativeness and justifying their exclusion from further processing.

The application of the Karhunen–Loève Transform provides decorrelation of signal components, reduces information redundancy, and forms an optimal representation of the process in the basis of principal components. This is achieved through the construction of the covariance matrix, computation of its eigenvalues and eigenvectors, and transformation into the optimal principal component basis.

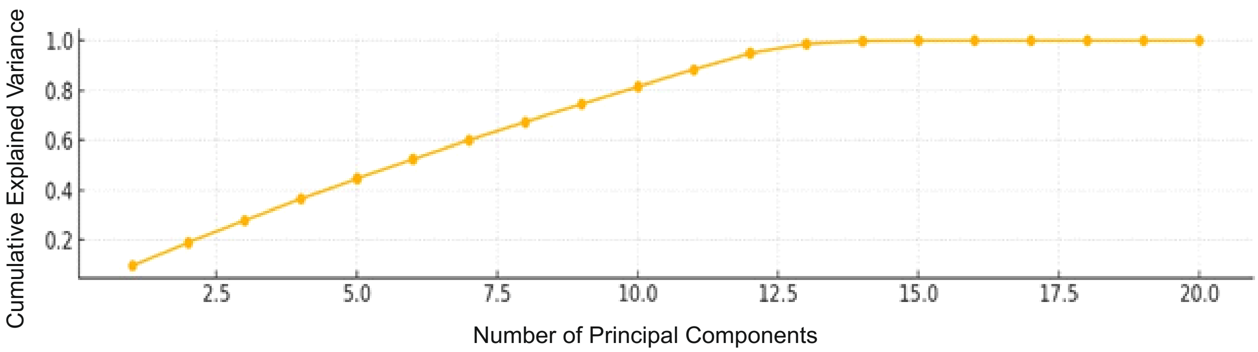

Figure 7 – Eigenvalue distribution of the covariance matrix

Therefore, for efficient analysis of a stochastic signal, it is sufficient to consider only those components that account for 95% of the total energy. This makes it possible to reduce data dimensionality without significant loss in the quality of signal reconstruction.

As shown in Fig. 8, the signal reconstructed using 10 principal components closely approximates the shape of the original input signal. The main characteristics of the signal, such as its energy structure and stochastic fluctuations, are preserved. Although the overall appearance of the signal after dimensionality reduction remains nearly identical to the input, minor local deviations are observed. This confirms the effectiveness of the Karhunen–Loève Transform for signal dimensionality reduction without significant loss of information, which is a crucial advantage for further optimization of stochastic signal processing.

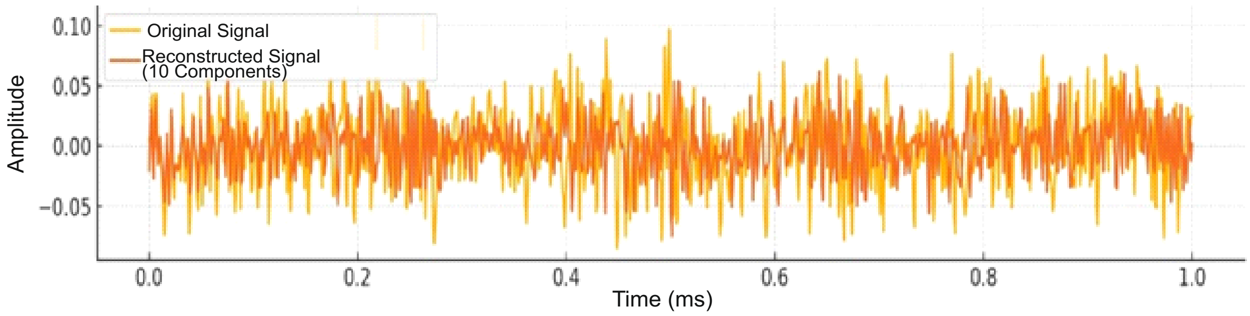

Figure 8 – Signal reconstruction using principal components

Based on the results of signal analysis using the Karhunen–Loève Transform and the determination of the required number of principal components containing at least 90% of the cumulative signal energy, the process of signal structure optimization is initiated through nonlinear processing in a reduced-dimensional space.

Accordingly, a second-order Volterra model is constructed using only the selected principal components. In this model, the memory depth M is also taken into account. The main objective of this stage is to achieve an effective combination of decorrelation and nonlinear modeling techniques [23]. The optimization of the Volterra kernel parameters is performed by minimizing spectral redundancy or the mean-square error in the principal component space using an adaptive weight adjustment procedure, which ensures maximum energy concentration within a narrow spectral range.

After completing signal processing in the principal component space and optimizing the Volterra kernel parameters, the signal is transformed back into the time domain by combining the principal components with the corresponding eigenvectors (basis functions) obtained from the Karhunen–Loève Transform, as defined in (7):

$$\hat{x}(n) = \sum_{k=1}^{M} \widehat{z_k}(n) \cdot e_k, \quad (7)$$

where $\hat{x}(n)$ – reconstructed signal;

$\widehat{z_k}$ – value of the processed k-th component at the n-th sample;

e_k – eigenvector (basis function) corresponding to the k-th principal component;

M – number of retained principal components.

The application of (7) enables the reconstruction of the signal from its optimized low-dimensional representation with reduced redundancy. As a result, a more compact and spectrally efficient signal in the time domain is obtained.

Fig. 9 shows the reconstructed signal after nonlinear processing in the principal component space using a second-order Volterra model. The upper plot represents the input centered and normalized signal, while the lower plot shows the reconstructed signal after processing in the principal component space.

As can be seen from the figure, the reconstructed signal preserves the main features of the original signal, while minor fluctuations are smoothed out. This indicates effective reduction of spectral redundancy without loss of essential information.

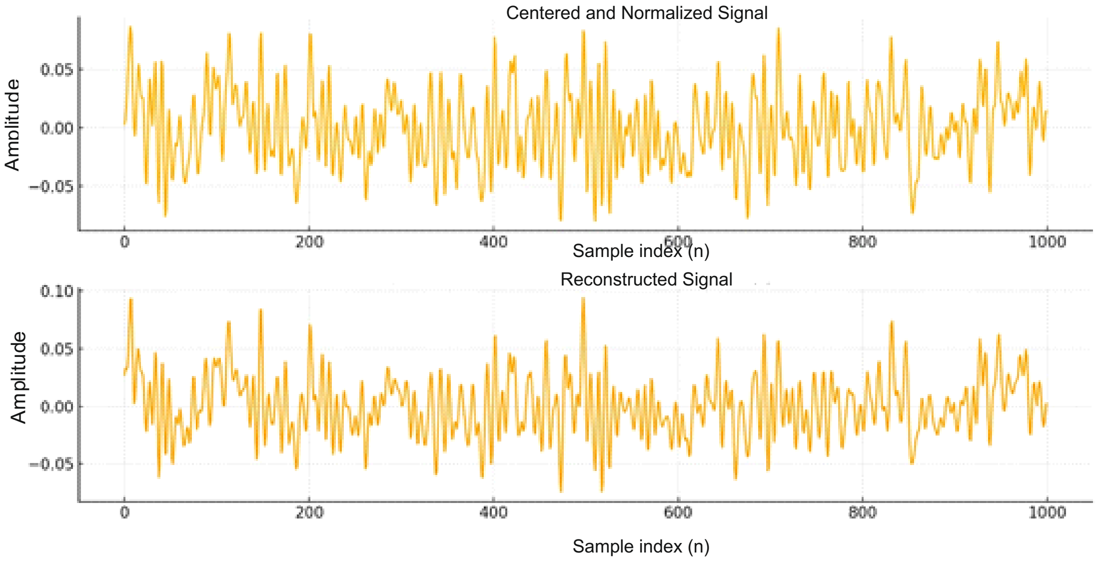

Figure 9 – Reconstructed signal after nonlinear processing in the principal component space

The final stage involves evaluating the spectral efficiency of the reconstructed signal based on changes in the power spectral density. This evaluation makes it possible to assess the achieved reduction in spectral bandwidth and the improvement in energy compactness.

Analysis of Fig. 10 shows that, after signal processing using the combination of the Karhunen–Loève Transform and Volterra series, the signal energy becomes concentrated within a narrower frequency band. In addition, a reduction in spectral redundancy is observed, indicating improved spectral efficiency.

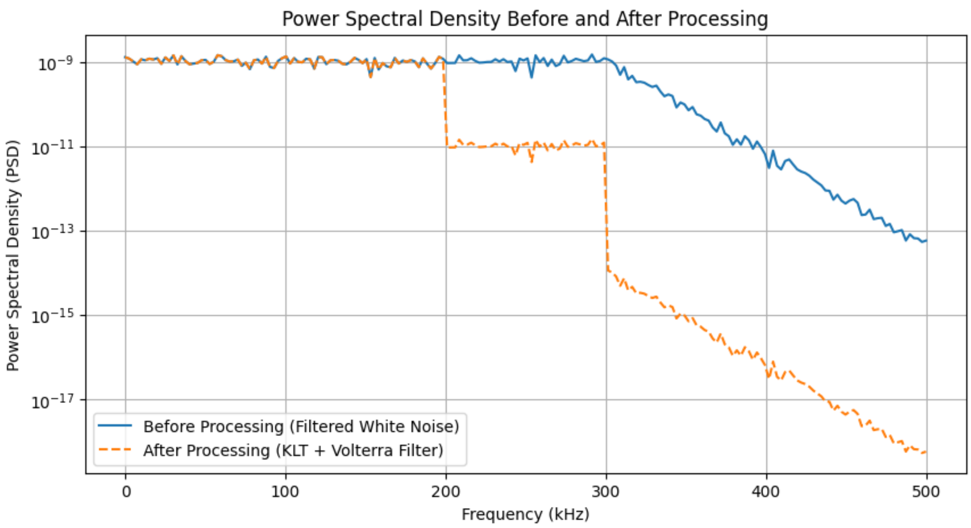

Figure 10 – Comparison of the power spectral density

As shown in Fig. 11, to preserve 90% of the signal energy, it is sufficient to retain only the first 10 principal components, since the contribution of the remaining components is negligible. This indicates the feasibility of discarding higher-order components in order to reduce signal dimensionality without significant loss of information.

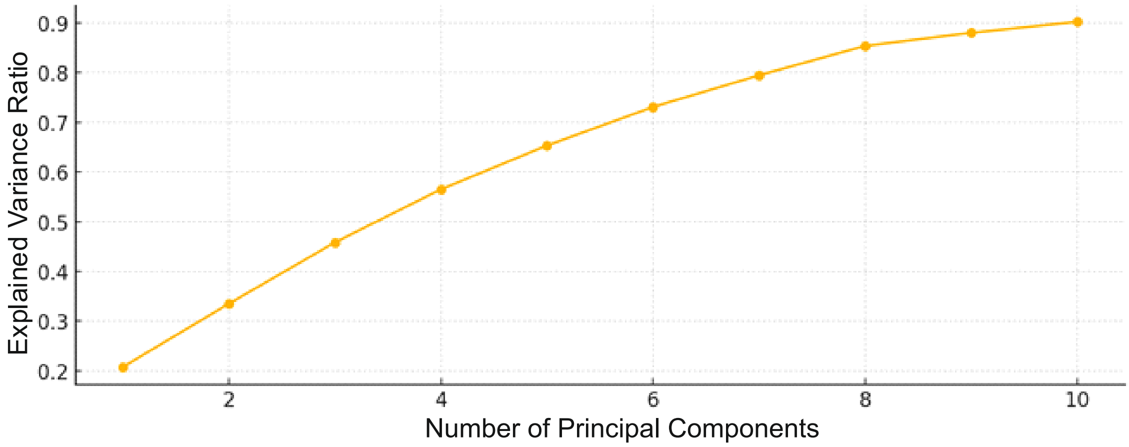

Figure 11 – Cumulative energy fraction in principal components

Fig. 12 presents the signal spectra before and after processing. As can be observed, the processed signal exhibits a lower amplitude but a smoother spectral shape with less pronounced high-frequency components. This confirms the reduction of spectral redundancy and the improvement of energy compactness while preserving the overall signal structure.

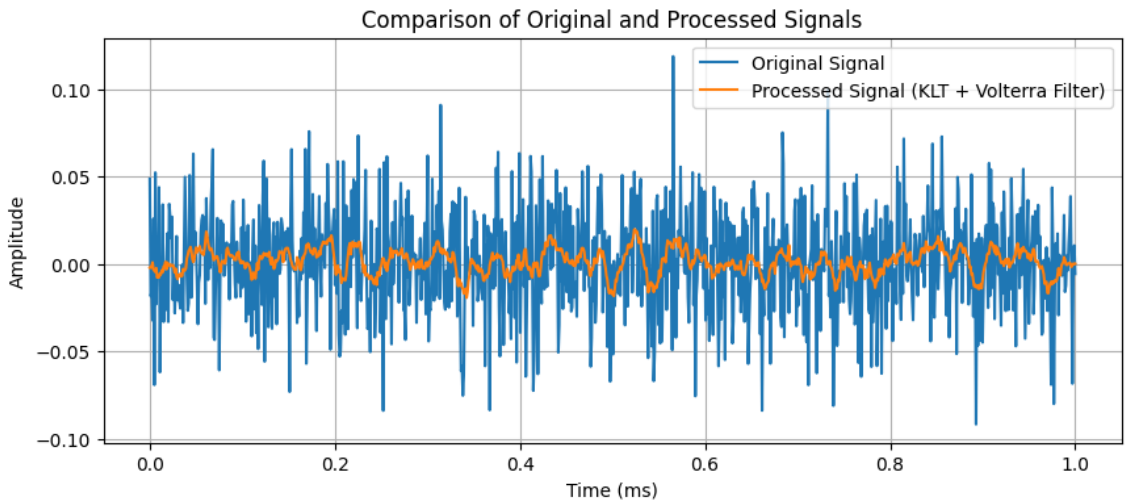

Figure 12 – Comparison of the original and processed signal spectra

The summarized results are presented in Table 1.

Table 1 – Comparison of spectral efficiency metrics of the signal before and after processing

№	Metric	Before processing	After processing	Change (%)
1.	Spectral bandwidth	300	~270	–10
2.	Spectral entropy (rel. units)	5.89	5,23	–11.2
3.	Energy fraction in 10 principal components	~0.9	~0,9	–
4.	Spectral redundancy (normalized)	1.00	0,89	–11
5.	Spectral efficiency gain			≈ +11%

Analysis of Table 1 indicates that the application of the combined signal processing method – namely, the Karhunen-Loève Transform in conjunction with a second-order Volterra model – enables a moderate but stable improvement in spectral efficiency. The reduction in spectral bandwidth by approximately 10%, along with the decrease in spectral entropy and normalized spectral redundancy, indicates that the signal energy becomes more concentrated within a narrower frequency range with fewer redundant components. At the same time, the preservation of a high energy fraction within the first 10 principal components confirms that the processing does not result in significant loss of informative signal content. Therefore, the proposed method achieves a balance between compression, decorrelation, and reconstruction accuracy of the stochastic signal.

Although the application of the algorithm for optimizing the spectral characteristics of stochastic signals through the combination of Volterra series and the Karhunen-Loève Transform enables positive improvements in spectral efficiency, numerical simulations indicates that the operation of modern information systems is influenced by various types of interference. These include, in particular, spoofing systems as well as dynamic changes in the radio frequency environment, which can significantly affect the performance of the algorithm.

Thus, signal processing in real-world conditions requires robustness of the applied methods to interference with varying spectral structures, including impulsive and adaptive interference, as well as fluctuations in frequency characteristics. Therefore, it is necessary to evaluate the robustness of the proposed methods to interference and their ability to adapt to dynamically changing environmental conditions.

According to experimental data [23], in the presence of noise interference, the spectral bandwidth increases on average by 15–35%, which proportionally reduces spectral efficiency.

The results reported in [4] show that, during interference modeling in FMCW radar systems, static methods exhibit a decrease in the percentage of correctly decoded signals, while the spectral bandwidth increases on average by 18–25%. In addition, under broadband jamming conditions, a reduction of the useful transmission bandwidth by 20–30% is observed, which directly affects spectral efficiency.

As shown in Fig. 13, under nominal conditions (i.e., in the absence of interference), the spectral efficiency of the system reaches 100%, which serves as a baseline reference level. In the presence of interference and without adaptive processing, spectral efficiency decreases to 76%, indicating a loss of nearly one-quarter of the transmitted information or an effective expansion of the occupied bandwidth. In contrast, the application of adaptive signal processing methods (such as modification of the Volterra kernel structure or adaptation of the KLT basis) allows the spectral efficiency to be restored up to 92%, representing a significant improvement compared to the fixed approach. Thus, the figure clearly demonstrates the necessity of incorporating adaptive mechanisms into signal processing algorithms, particularly under conditions of a dynamic radio-frequency environment.

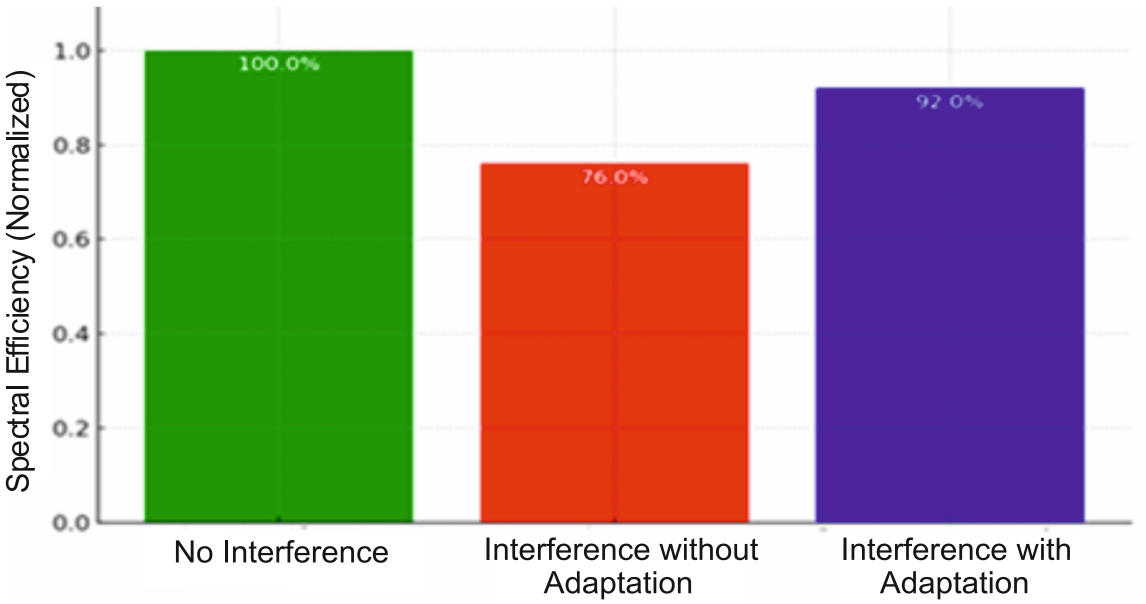

Figure 13 – Comparison of spectral efficiency under interference conditions

Fig. 14 illustrates the variation of spectral efficiency depending on the signal processing method (fixed, KLT, KLT+Volterra) and the interference scenario.

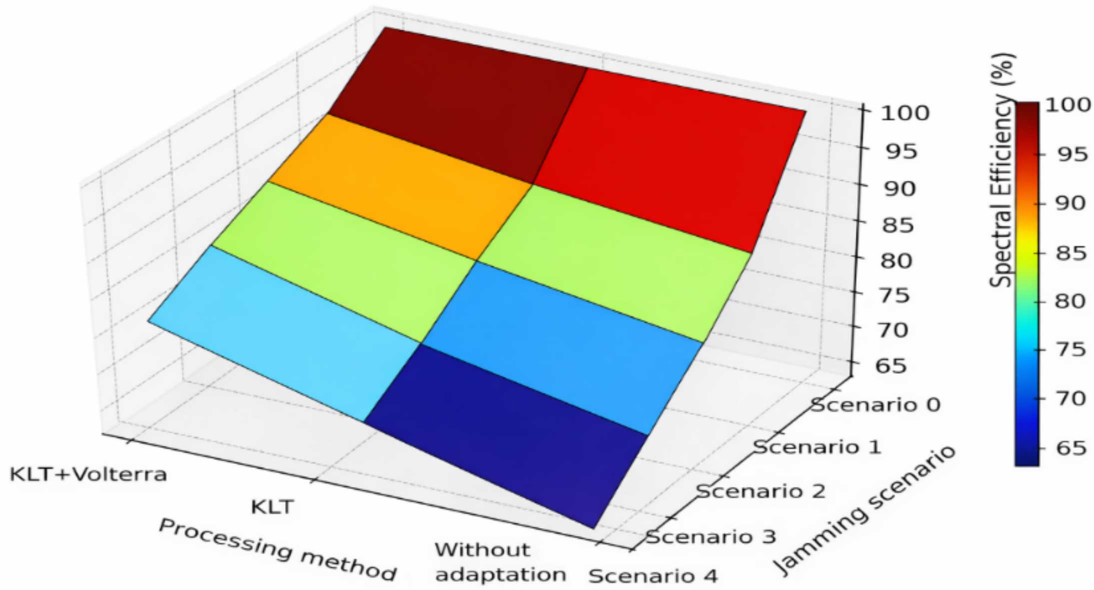

Figure 14 – Comparison of signal processing methods under interference conditions

An interference scenario is a generalized concept that describes a specific situation or conditions of interference impact on a signal processing system. In the context of modeling, each scenario represents a set of parameters that characterize the type, intensity, and structure of interference arising in the communication environment.

In this study, the interference scenarios are defined as follows:

Scenario 0 – nominal conditions, no interference (reference case).

Scenario 1 – low-frequency jamming (an example of active electronic warfare, EW).

Scenario 2 – impulsive interference with random distribution.

Scenario 3 – wideband noise jamming and spoofing.

Scenario 4 – combined attack: frequency fluctuations and adaptive signal imitation.

Such a classification enables a systematic evaluation of algorithm robustness under different types of signal interference. Typically, each scenario is implemented as a software simulation or an emulated environment with varying signal-to-noise ratios, spectral shifts, and modulation characteristics of the interference. It is evident that as the intensity and complexity of interference increase (from Scenario 0 to Scenario 4), fixed methods lose stability, with efficiency dropping to 63%, whereas the combined method maintains a performance level above 80%. This demonstrates the robustness of the adaptive approach under complex interference conditions.

The generalized simulation results are presented in Table 2, which illustrates the dependence of spectral efficiency on the signal processing method and the complexity of the interference scenario. Additionally, a brief description of each scenario is provided to facilitate the interpretation of the testing conditions.

Table 2 – Comparison of spectral efficiency (%) for different interference scenarios and signal processing methods

Interference Scenario	Scenario Description	Without Adaptation	KLT	KLT + Volterra
Scenario 0	No interference	100	100	100
Scenario 1	Low-frequency jamming	85	89	93
Scenario 2	Impulsive interference	78	84	89
Scenario 3	Wideband jamming + spoofing	70	77	85
Scenario 4	Combined attack with frequency fluctuations	63	72	80

Thus, the analysis of interference impact demonstrates that conventional signal processing methods rapidly lose effectiveness as the complexity of interference increases. At the same time, the adaptive approach based on the combination of orthogonal decomposition and nonlinear modeling enables the preservation of stable spectral characteristics even under combined interference conditions. The application of this method significantly reduces sensitivity to structurally correlated and spectrally non-stationary interference, as confirmed by simulations across multiple scenarios. This highlights the potential of the proposed algorithm for application in information systems operating within dynamically unstable radio-frequency environments.

Conclusions and prospects for further research.

This paper proposes a hybrid method for stochastic signal formation that combines the Karhunen–Loève Transform with nonlinear Volterra modeling. The proposed approach performs nonlinear processing in the principal-component domain, providing statistical decorrelation, energy concentration, and improved matching between the nonlinear model and the signal structure.

Numerical simulations demonstrate that the method improves the spectral characteristics of structured stochastic signals. Compared with conventional processing, the proposed algorithm reduces the effective spectral bandwidth by approximately 10%, decreases spectral entropy and normalized spectral redundancy by about 11%, and increases spectral efficiency by approximately 11%, while preserving nearly 90% of the signal energy within the first ten principal components. For broadband noise-like signals, however, nonlinear processing without preliminary statistical decorrelation may reduce spectral efficiency.

The study also establishes the practical limitations of the proposed approach. Its performance depends on the statistical properties of the processed signal, the selected number of principal components, and the tuning of the Volterra model parameters. Consequently, adaptive parameter selection is required for reliable operation under dynamically changing interference conditions.

Overall, the obtained results confirm that combining orthogonal statistical decomposition with nonlinear modeling is an effective approach for controlled stochastic signal synthesis with improved spectral efficiency and reduced computational complexity, making it suitable for communication, control, and radar systems operating under limited radio-frequency resources.

References

1. Loève, M. Probability Theory I. 4th ed. Vol. 45. New York: Springer-Verlag, 1977. 428 p. https://doi.org/10.1007/978-1-4684-9464-8.
2. Jolliffe I. T., Cadima J. Principal component analysis: A review and recent developments. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 2016, 374(2065), Article 20150202. https://doi.org/10.1098/rsta.2015.0202.
3. Chang H., Wang C., Liu Z., Feng B., Zhan C., Cheng X. Research on the Karhunen–Loève transform method and its application to hull form optimization. Journal of Marine Science and Engineering, 2023, 11(1), Article 230. https://doi.org/10.3390/jmse11010230.
4. Zhu L., Liu Y., He D., Guan K., Ai B., Zhong Z., Liao X. An efficient target detection algorithm via Karhunen–Loève transform for frequency modulated continuous wave (FMCW) radar applications. IET Signal Processing, 2022, 16(7), 800–810. https://doi.org/10.1049/sil2.12111.
5. Maccone C. Advantages of Karhunen–Loève transform over fast Fourier transform for planetary radar and space debris detection. Acta Astronautica, 2007, 60(8–9), 775–779. https://doi.org/10.1016/j.actaastro.2006.08.015.
6. Orcioni S. Improving the approximation ability of Volterra series identified with a cross-correlation method. Nonlinear Dynamics, 2014, 78, 2861–2869. https://doi.org/10.1007/s11071-014-1631-7.
7. Stenger A., Rabenstein R. Adaptive Volterra filters for acoustic echo cancellation. In Proceedings of the IEEE-EURASIP Workshop on Nonlinear Signal and Image Processing (NSIP'99), Vol. 2, Antalya, Turkey, 20–23 June 1999, pp. 679–683.
8. Kosambi D. D. Statistics in function space. Journal of the Indian Mathematical Society, 1943, 7, 76–88.
9. Ghanem R. G., Spanos P. D. Stochastic Finite Elements: A Spectral Approach. New York: Springer-Verlag, 1991. https://doi.org/10.1007/978-1-4612-3094-6.
10. Schenk C. A., Schuëller G. I. Uncertainty Assessment of Large Finite Element Systems. Lecture Notes in Applied and Computational Mechanics, Vol. 24. Berlin, Heidelberg: Springer, 2005. https://doi.org/10.1007/11673941.
11. Chowdhary K., Najm H. N. Bayesian estimation of Karhunen–Loève expansions: A random subspace approach. Journal of Computational Physics, 2016, 319, 280–293. https://doi.org/10.1016/j.jcp.2016.02.056.
12. Kober V., Makovetskii A., Voronin S. Karhunen–Loève expansion using a parametric model of oscillating covariance function. Mathematics, 2025, 13(16), Article 2569. https://doi.org/10.3390/math13162569.
13. Gariachiy M., Shcherbinin S. Method for formation of stochastic signals using Volterra series. Measuring and Computing Devices in Technological Processes, 2026, No. 1, 47–60. https://doi.org/10.31891/2219-9365-2026-85-7.
14. Jones I. F., Levy S. Signal-to-noise ratio enhancement in multichannel seismic data via the Karhunen–Loève transform. Geophysical Prospecting, 1987, 35(1), 12–32.
15. Cohen I., Berdugo B. Speech enhancement for non-stationary noise environments. Signal Processing, 2001, 81(11), 2403–2418.
16. Díez-García R., Camps A. Adaptive near real-time RFI mitigation using Karhunen–Loève transform. Remote Sensing, 2025, 17(15), Article 2578. https://doi.org/10.3390/rs17152578.

17. Mallat S. A Wavelet Tour of Signal Processing: The Sparse Way. 3rd ed. Burlington, MA: Academic Press/Elsevier, 2009.
18. Rugh W. J. Nonlinear System Theory: The Volterra/Wiener Approach. Baltimore, MD: The Johns Hopkins University Press, 1981. 336 p.
19. Smirnov O. A., Dorenskyi O. P., Drieiev O. M. Analysis of image compression and reconstruction processes based on digital methods. Science and Technology of the Air Force of Ukraine, 2013, 3(12), 122–127.
20. Kapustii B. O., Rusyn B. P., Taninov V. A. Criteria for optimization of a set of spectral components of the Karhunen–Loève transform in calculating the differential probability of correct recognition. Radio Electronics, Computer Science, Control, 2004, No. 3, 118–123.
21. Oletskyi O. V. Algorithm for reconstruction of one-dimensional signals based on the integral Karhunen–Loève expansion. NaUKMA Research Papers. Computer Science, 2002, 19–20, 74–76.
22. Gariachiy M., Shcherbinin S. Analysis of methods for enhancing spectral efficiency in information systems. Science-Based Technologies, 2025, 67(3), 325–339. https://doi.org/10.18372/2310-5461.67.18510.
23. Ipatov V. P. Spread Spectrum and CDMA: Principles and Applications. Chichester: John Wiley & Sons, 2005. Print ISBN 978-0-470-09178-4; Online ISBN 978-0-470-09180-7. https://doi.org/10.1002/0470091800.

Історія статті:
Отримано: 27.05.2026 Доопрацьовано: 09.09.2026 Прийнято до друку: 23.09.2026 Опубліковано: 29.09.2026