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ASSESSMENT OF STRUCTURAL EFFICIENCY OF UNEQUAL-ENERGY COMPLEX SIGNAL ENSEMBLES IN ULTRA-WIDEBAND CODE-DIVISION MULTIPLE ACCESS SYSTEMS

Zhuchenko O. Assessment of Structural Efficiency of Unequal-Energy Complex Signal Ensembles in Ultra-Wideband Code-Division Multiple Access Systems. Selecting unequal-energy complex-signal ensembles for ultra-wideband code-division multiple-access systems requires evaluating their structural efficiency with respect to multiple-access interference. Exact comparison based on individual interference energies requires calculations for every user, resulting in quadratic computational complexity with increasing ensemble size. This paper proposes a structural efficiency metric defined as the product of the structural equivalent ensemble size and the squared maximum cross-correlation within the ensemble. Based on this metric, a method is developed for comparing and ranking unequal-energy complex-signal ensembles of different sizes without calculating individual multiple-access interference energies. The proposed method is evaluated against the maximum individual normalized multiple-access interference energy and the normalized energy-weighted Total Squared Correlation functional. For ensembles of the same size, both methods show almost complete agreement with the individual interference-based comparison. For ensembles of different sizes, the proposed metric agrees with the reference comparison substantially more often than the normalized energy-weighted Total Squared Correlation functional. The proposed method has linear computational complexity, compared with quadratic complexity for both reference approaches. The results demonstrate that the structural equivalent ensemble size combined with the maximum cross-correlation provides an efficient conservative metric for comparing, ranking, and selecting unequal-energy complex-signal ensembles.
Keywords: structural efficiency; unequal-energy complex signals; structural equivalent ensemble size; multiple-access interference; code-division multiple access; ensemble ranking.

Жученко О. С. Оцінка структурної ефективності ансамблів різноенергетичних складних сигналів надширокосмугових систем множинного доступу з кодовим розділенням. Вибір ансамблів різноенергетичних складних сигналів для надширокосмугових систем множинного доступу з кодовим розділенням потребує оцінювання їх структурної ефективності з урахуванням завад множинного доступу. Точне порівняння на основі індивідуальних енергій завад потребує розрахунків для кожного користувача, що зумовлює квадратичну обчислювальну складність зі збільшенням об'єму ансамблю. У роботі запропоновано показник структурної ефективності, визначений як добуток структурного еквівалентного об'єму ансамблю та квадрата максимального значення взаємної кореляції в ансамблі. На основі цього показника розроблено метод порівняння та ранжування ансамблів різноенергетичних складних сигналів різного об'єму без обчислення індивідуальних енергій завад множинного доступу. Запропонований метод оцінено шляхом порівняння з найбільшим значенням індивідуальної нормованої енергії завад множинного доступу та нормованим енергетично-зваженим функціоналом загальної квадратичної кореляції. Для ансамблів однакового об'єму обидва методи демонструють практично повний збіг із результатами порівняння за індивідуальними завадами. Для ансамблів різного об'єму запропонований показник значно частіше узгоджується з результатами еталонного порівняння, ніж нормований енергетично-зважений функціонал загальної квадратичної кореляції. Запропонований метод має лінійну обчислювальну складність порівняно з квадратичною складністю обох еталонних підходів. Результати підтверджують, що структурний еквівалентний об'єм у поєднанні з максимальним значенням взаємної кореляції забезпечує ефективний консервативний показник для порівняння, ранжування та вибору ансамблів різноенергетичних складних сигналів.
Ключові слова: структурна ефективність; різноенергетичні складні сигнали; структурний еквівалентний об'єм; завади множинного доступу; множинний доступ з кодовим розділенням; ранжування ансамблів.

Statement of a scientific problem.

The performance of code-division multiple-access systems depends significantly on the choice of a complex-signal ensemble. The ensemble determines the signal structure, including the set of signal components, their parameters and positions on the time axis within the signal duration, the maximum number of users through the ensemble size, and the maximum level of multiple-access interference experienced by one user from all other users. The structure of the complex signals directly affects multiple-access interference and is characterized by signal correlations and energies [1–4].

A promising approach to the development of code-division multiple-access systems is the use of pulsed complex-signal ensembles with minimal energy interaction. Such ensembles can be represented as complex time-division signal ensembles with a periodic structure, which more completely reflects their properties. A characteristic feature of these ensembles is their inherently unequal signal energies resulting from different numbers of pulses at the same individual pulse duration [5–8].

For equal-energy complex-signal ensembles, the effect of ensemble structure on multiple-access interference is determined by the ensemble size and cross-correlations between signals. For unequal-energy complex-signal ensembles, multiple-access interference depends on both cross-correlations and signal energies. Therefore, when evaluating the efficiency of unequal-energy complex-signal ensembles, the nonuniform distribution of signal energies represents an additional factor affecting multiple-access interference and, consequently, ensemble efficiency metrics.

The development of ultra-wideband code-division multiple-access systems requires a structural efficiency metric that provides a numerical measure of the effect of complex-signal ensemble structure on multiple-access interference, particularly for comparing, ranking, and selecting unequal-energy complex-signal ensembles.

Research analysis.

The basic principles of code-division multiple access relate multiple-access interference to user signal energies and cross-correlations between signals in the ensemble [1–4]. For unequal-energy complex-signal ensembles, the total multiple-access interference (MAI) energy depends simultaneously on signal cross-correlations and the nonuniform distribution of signal energies.

Based on the analysis of existing efficiency metrics for complex-signal ensembles, including the Total Squared Correlation (TSC) functional and its weighted variants [10–12], the following requirements for a structural efficiency metric were formulated in [9]: it should be related to the Signal-to-Interference-plus-Noise Ratio (SINR), be sensitive to unfavorable MAI conditions, and enable ranking and comparison of both equal- and unequal-energy complex-signal ensembles of different sizes.

For equal-energy complex-signal ensembles, structural efficiency can be evaluated using the cross-correlations between ensemble signals. For such ensembles, an appropriate structural efficiency metric is the product of the ensemble size, which provides an approximate estimate of the normalized total energy of interfering signals, and the squared maximum cross-correlation within the ensemble [9].

In [13, 14], an equivalent transformation of an unequal-energy complex-signal ensemble into a hypothetical equal-energy ensemble with preserved total signal energy was proposed. The set of signal energies is approximately reduced to a scalar quantity termed the structural equivalent ensemble size, whose numerical value represents an approximate estimate of the normalized total energy of interfering signals. Thus, the structural equivalent ensemble size allows efficiency metrics developed for equal-energy complex-signal ensembles to be extended to unequal-energy ensembles.

The purpose of this study. This study aims to define a structural efficiency metric for unequal-energy complex-signal ensembles based on the structural equivalent ensemble size and the maximum cross-correlation within the ensemble, and to develop a corresponding method for evaluating the structural efficiency of signal ensembles.

Presentation of the main material and substantiation of the obtained research results. In the absence of noise $E_{N,i} = 0$, the general expression for SINR is given by:

$$\mathrm{SINR}_i^{(\mathrm{E_{N,i}}=0)} = \frac{E_i}{E_{I,i}}. \quad (1)$$

The individual multiple-access interference energy $E_{I,i}$ is determined by the set of interfering-user signal energies and the corresponding cross-correlation values within the ensemble:

$$E_{I,i} = \sum_{j=1,\, j\neq i}^{L} E_j \left(R_{ij}^{(max)}\right)^2. \quad (2)$$

For an equal-energy complex-signal ensemble $E_j = E, j = 1, \dots, L$, the multiple-access interference energy for the i-th user is determined as:

$$E_{I,i} = E \sum_{j=1,\, j\neq i}^{L} \left(R_{ij}^{(max)}\right)^2. \quad (3)$$

In this case, the multiple-access interference energy $E_{I,i}$ no longer depends on the individual signal energies of particular users, since all signals in the ensemble have the same energy.

If, for the equal-energy signal ensemble as a whole, we assume:

$$R_{max}^2 = \max_{i,j,i\neq j}\left(R_{ij}^{(max)}\right)^2, \tag{4}$$

we obtain an upper bound $E_{I,i}^{(max)}$ on the total multiple-access interference energy:

$$E_{I,i}^{(max)} = (L-1)ER_{max}^2. \tag{5}$$

This bound is determined by the ensemble signal energy E, the number of interfering signals $L-1$, and the maximum squared cross-correlation R_{max}^2 within the ensemble. For an equal-energy complex-signal ensemble, in the absence of noise and using the upper bound $E_{I,i}^{(max)}$ on the interference energy, we obtain the following estimate $\mathrm{SINR}^{(E_{N,i}=0)}$:

$$\mathrm{SINR}^{(E_{N,i}=0)} = \mathrm{SINR}_{min} = \frac{1}{(L-1)R_{max}^2} = \frac{1}{\gamma_{\mathrm{str}}}, \tag{6}$$

where γ_{str} – the structural efficiency metric of the ensemble is defined as:

$$\gamma_{\mathrm{str}} = (L-1)R_{max}^2. \tag{7}$$

For sufficiently large ensemble sizes $L \gg 1$, the difference between the corresponding ensemble-size terms $L-1$ and L is relatively small. Therefore $L-1 \approx L$, the approximation used in the following expression is acceptable:

$$\gamma_{\mathrm{str}} = LR_{max}^2, \tag{8}$$

After applying this approximation, the expression for the structural efficiency estimate becomes:

$$\mathrm{SINR}^{(E_{N,i}=0)} = \mathrm{SINR}_{min} = \frac{1}{\gamma_{\mathrm{str}}}. \tag{9}$$

For unequal-energy complex-signal ensembles, we introduce a normalized quantity by dividing the individual multiple-access interference energy by the signal energy of the i-th user:

$$\gamma_{\mathrm{str},i} = \frac{E_{I,i}}{E_i} = \frac{\sum_{j=1,\,j\neq i}^{L} E_j\left(R_{ij}^{(max)}\right)^2}{E_i}. \tag{10}$$

We then use an upper bound on the individual multiple-access interference energy for the i-th user of an unequal-energy complex-signal ensemble $E_{I,i} \leq E_{I,i}^{(max)}$:

$$E_{I,i}^{(max)} = R_{max}^2(E_\Sigma - E_i). \tag{11}$$

Since the signal energy of the i-th user is positive, normalization of the upper bound preserves the direction of the inequality:

$$\gamma_{\mathrm{str},i} = \frac{E_{I,i}}{E_i}, \quad \gamma_{\mathrm{str},i}^{(max)} = R_{max}^2\frac{(E_\Sigma - E_i)}{E_i}, \quad \gamma_{\mathrm{str},i} \leq \gamma_{\mathrm{str},i}^{(max)}, \tag{12}$$

$$\gamma_{\mathrm{str},i} \leq R_{max}^2\frac{(E_\Sigma - E_i)}{E_i}. \tag{13}$$

Thus, for an unequal-energy complex-signal ensemble, the structural efficiency quantity $\gamma_{str,i}$ for an individual user represents the ratio of multiple-access interference energy to the signal energy of that user, while the right-hand side of the inequality provides its upper bound in terms of the maximum cross-correlation within the ensemble. In the absence of noise $E_{N,i} = 0$, this quantity is the reciprocal of the corresponding $\mathrm{SINR}^{(E_{N,i}=0)}$.

$$\mathrm{SINR}_i^{(\mathrm{E_{N,i}}=0)} = \frac{E_i}{E_{I,i}} = \frac{1}{\gamma_{\mathrm{str},i}}. \tag{14}$$

Since the SINR and the individual structural efficiency quantity are reciprocal in the absence of noise, the minimum SINR among all users $\mathrm{SINR}_{min}^{(\mathrm{E_{N,i}}=0)}$ corresponds to the maximum value of the structural efficiency quantity $\gamma_{\mathrm{str},i}$. Therefore, the structural efficiency metric of the ensemble is determined by the worst-case user.

$$\gamma_{\mathrm{str}} = \max_{1\le i\le L} \gamma_{\mathrm{str,i}}. \tag{15}$$

In the absence of noise, the minimum SINR is thus determined by the reciprocal of the ensemble structural efficiency metric.

$$\mathrm{SINR}_{min}^{(\mathrm{E_{N,i}}=0)} = \min_{1\le i\le L} \frac{E_i}{E_{I,i}} = \frac{1}{\gamma_{\mathrm{str}}}. \tag{16}$$

For an equal-energy complex-signal ensemble, $E_i = E, i = 1, \dots, L$, the total energy of the ensemble signals is $E_\Sigma = \sum_{k=1}^{L} E_k = LE$, and the right-hand side of the estimate $\gamma_{\mathrm{str},i} \le R_{max}^2 (E_\Sigma - E_i)/E_i$ takes the same value for all users:

$$R_{max}^2 \frac{E_\Sigma - E_i}{E_i} = R_{max}^2 \frac{E_\Sigma - E_i}{E_i} = R_{max}^2 \frac{LE - E}{E} = R_{max}^2 (L-1). \tag{17}$$

Therefore, the estimate of $\gamma_{\mathrm{str},i}$ takes the form:

$$\gamma_{\mathrm{str},i} \le R_{max}^2 (L-1). \tag{18}$$

Since the right-hand side of the inequality $\gamma_{\mathrm{str},i} \le R_{max}^2 (L-1)$ does not depend on the user index, the maximum value of $\gamma_{\mathrm{str,i}}$ over all users defines a conservative structural efficiency metric for an equal-energy complex-signal ensemble:

$$\gamma_{\mathrm{str}} = (L-1) R_{max}^2, \tag{19}$$

or, using the approximation $L - 1 \approx L$ for $L \gg 1$:

$$\gamma_{\mathrm{str}} = L R_{max}^2. \tag{20}$$

The structural equivalent ensemble size $L_{\mathrm{eq}}^{(+)}$ is introduced as the effective number of equal-energy signals that replaces the energy-related component L in the structural efficiency metric for an unequal-energy complex-signal ensemble:

$$L_{\mathrm{eq}}^{(+)} = L^2 \frac{\sum_{i=1}^{L} E_i^2}{\left(\sum_{i=1}^{L} E_i\right)^2}. \tag{21}$$

For an equal-energy complex-signal ensemble, the structural equivalent ensemble size $L_{\mathrm{eq}}^{(+)}$ coincides with the size of the original ensemble L . As the nonuniformity of the signal energy distribution increases, the structural equivalent ensemble size $L_{\mathrm{eq}}^{(+)}$ increases monotonically.

For an unequal-energy complex-signal ensemble, the structural efficiency metric is defined by replacing the energy-related component L with the structural equivalent ensemble size $L_{\mathrm{eq}}^{(+)}$ while retaining the correlation-related component R_{max}^2 :

$$\gamma_{\text{str}}^{(+)} = L_{\text{eq}}^{(+)} R_{max}^2. \tag{22}$$

Based on the proposed structural efficiency metric $\gamma_{\text{str}}^{(+)}$, an algorithm for evaluating the structural efficiency of unequal-energy complex-signal ensembles is developed.

For each of the M signal ensembles being compared, $s = 1, \dots, M$, the required input data include the ensemble size L_s , the set of signal energies $E_{i,s}, i = 1, \dots, L_s$, the value of $R_{max,s}^2$, or the corresponding data required for its exact calculation or approximate estimation.

The steps of the algorithm for evaluating the structural efficiency of unequal-energy complex-signal ensembles, which can be used for selecting, comparing, or ranking signal ensembles, are given below.

Step 1. Specify the input data:

1. number of signal ensembles being compared M;
2. index of the ensemble being compared $s = 1, \dots, M$;
3. ensemble size L_s;
4. set of signal energies $E_{i,s},\ i = 1, \dots, L_s$;
5. squared maximum cross-correlation of the ensemble $R_{max,s}^2$, or the corresponding ensemble parameters required for its calculation or approximate estimation.

Step 2. Determine the squared maximum cross-correlation for each signal ensemble $R_{max,s}^2$, $s = 1, \dots, M$:

$$R_{max,s}^2 = \max_{1 \le i < j \le L_s} \left(R_{ij,s}^{(max)} \right)^2. \tag{23}$$

For complex time-division signal ensembles with a periodic structure, the squared maximum cross-correlation $R_{max,s}^2$ is determined by the pair of signals with the minimum product of their energies:

$$R_{max,s}^2 = \frac{1}{\min\limits_{1 \le i < j \le L_s} E_{i,s} \cdot E_{j,s}}. \tag{24}$$

Step 3. Calculate the structural equivalent ensemble size for each signal ensemble:

$$L_{\text{eq},s}^{(+)} = L_s^2 \frac{\sum_{i=1}^{L_s} E_{i,s}^2}{\left(\sum_{i=1}^{L_s} E_{i,s} \right)^2}. \tag{25}$$

The quantity $L_{\text{eq},s}^{(+)}$ reflects the increase in multiple-access interference energy experienced by users with the lowest signal energies due to the dominance effect of users with higher signal energies.

Step 4. Calculate the structural efficiency metric for each signal ensemble:

$$\gamma_{\text{str},s}^{(+)} = L_{\text{eq},s}^{(+)} R_{max,s}^2. \tag{26}$$

The quantity $\gamma_{\text{str},s}^{(+)}$ provides a conservative estimate of the structural efficiency of the ensemble because the structural equivalent ensemble size $L_{\text{eq},s}^{(+)}$ accounts for the nonuniform distribution of signal energies within the ensemble and reflects the increase in multiple-access interference for users with lower signal energies, while $R_{max,s}^2$ represents the worst-case cross-correlation within the complex-signal ensemble.

Step 5. Compare, select, or rank the ensembles according to the structural efficiency metric $\gamma_{\text{str},s}^{(+)}$, $s = 1, \dots, M$, by arranging the metric values for the M ensembles being compared in ascending order:

1. set of structural efficiency metric values $\gamma_{\text{str},s}^{(+)}$, $s = 1, \dots, M$, before ordering:

$$\gamma_{\text{str},1}^{(+)}, \dots, \gamma_{\text{str},s}^{(+)}, \gamma_{\text{str},s+1}^{(+)}, \dots, \gamma_{\text{str},M}^{(+)};$$

2. set of structural efficiency metric values $\gamma_{\text{str},s_m}^{(+)}$ arranged in ascending order:

$$\gamma_{\text{str},s_1}^{(+)} \le \dots \le \gamma_{\text{str},s_m}^{(+)} \le \gamma_{\text{str},s_{m+1}}^{(+)} \le \dots \le \gamma_{\text{str},s_M}^{(+)},$$

where s_m is the original index of the ensemble that occupies the m-th position after ordering;

m is the ensemble position after ordering in ascending order of the structural efficiency metric $\gamma_{\text{str},s_m}^{(+)}$, $m = 1, \dots, M$.

The ensemble with index s_1, whose structural efficiency metric $\gamma_{\text{str},s_1}^{(+)}$ occupies the first position $m = 1$ in the ascending ordered set, is the best according to the structural efficiency metric. The ensemble with index s_M, whose metric value $\gamma_{\text{str},s_M}^{(+)}$ occupies the last position $m = M$ in this set, is the worst according to the structural efficiency metric.

If the elements of the ordered sequence at positions m and $m+1$ satisfy the equality $\gamma_{\text{str},s_m}^{(+)} = \gamma_{\text{str},s_{m+1}}^{(+)}$, then the ensembles with indices s_m and s_{m+1} are equivalent according to the structural efficiency metric.

Step 6. Output the results:

1. the sequence of ensemble indices arranged in ascending order of the structural efficiency metric:

$$s_1, \dots, s_m, s_{m+1}, \dots, s_M;$$

2. the set of structural efficiency metric values arranged in ascending order:

$$\gamma_{\text{str},s_1}^{(+)} \le \dots \le \gamma_{\text{str},s_m}^{(+)} \le \gamma_{\text{str},s_{m+1}}^{(+)} \le \dots \le \gamma_{\text{str},s_M}^{(+)}.$$

Next, the ensemble ranking results obtained using the structural efficiency metric will be compared with those obtained using the ratio of individual multiple-access interference energy to useful signal energy. For each ensemble, the individual multiple-access interference energies $E_{I,i}, i = 1, \dots, L$ and the corresponding quantities $\gamma_{\text{str},i} = \frac{E_{I,i}}{E_i}$ will be calculated. The numerical estimate of ensemble structural efficiency based on the direct calculation of individual multiple-access interference energies is defined as the maximum of the individual values $\gamma_{\text{str}} = \max_{1 \le i \le L} \gamma_{\text{str},i}$.

Additionally, the ensembles will be compared using the normalized energy-weighted Total Squared Correlation functional TSC_{enwt}:

$$\text{TSC}_{\text{enwt}} = \frac{\text{TSC}_{\text{E}}}{E_\Sigma^2} = \frac{1}{E_\Sigma^2} \sum_{i=1}^{L} \sum_{j=1}^{L} E_i \, E_j \left(R_{ij}^{(max)}\right)^2. \quad (27)$$

For further comparison of the ensembles, three estimates are used:

1. The estimate based on the maximum individual relative level of multiple-access interference γ_{str}, obtained after calculating the individual multiple-access interference energies $E_{I,i}$ and the corresponding quantities $\gamma_{\text{str},i} = E_{I,i}/E_i$;

2. The estimate based on the structural efficiency metric $\gamma_{\text{str}}^{(+)} = L_{\text{eq}}^{(+)} R_{max}^2$;

3. The estimate based on the normalized energy-weighted Total Squared Correlation functional TSC_{enwt}.

For all three estimates, a lower numerical value corresponds to a better ensemble.

For complex time-division signals with a periodic structure, Figs. 1–3 present the results of ensemble comparison using the estimate γ_{str}, the structural efficiency metric $\gamma_{\text{str}}^{(+)}$, and the normalized energy-weighted functional TSC_{enwt} with $\gamma_{\text{str}} = LR_{max}^2$.

For each pair of ensembles, the comparison result based on the individual multiple-access interference energies was determined first. For the same pair, the comparison results based on the structural efficiency metric $\gamma_{\text{str}}^{(+)}$ and the normalized energy-weighted functional TSC_{enwt} were then determined. Finally, it was checked whether the result obtained by each of these two methods agreed with the reference result.

The following events were considered in the comparison.

Event 1 corresponded to the case in which the results obtained using $\gamma_{\text{str}}^{(+)}$ and TSC_{enwt} both agreed with the result obtained using γ_{str}. Event 2 corresponded to the case in which the result obtained using $\gamma_{\text{str}}^{(+)}$ agreed with the result obtained using γ_{str}, whereas the result obtained using TSC_{enwt} differed. Event 3 corresponded to the case in which the result obtained using TSC_{enwt} agreed with the result obtained using γ_{str}, whereas the result obtained using $\gamma_{\text{str}}^{(+)}$ differed. Event 4 corresponded to the case in which the results obtained using both $\gamma_{\text{str}}^{(+)}$ and TSC_{enwt} differed from the result obtained using γ_{str}.

The advantage of the structural efficiency metric $\gamma_{\text{str}}^{(+)}$ corresponded to Event 2, since in this case the comparison result obtained using $\gamma_{\text{str}}^{(+)}$ agreed with that based on the individual multiple-access interference energies, whereas the result obtained using TSC_{enwt} did not. The frequencies of all four events were then calculated for the entire set of ensemble pairs.

Figure 1 shows the event frequencies for pairs of ensembles of the same size. Complete enumeration was performed for ensemble sizes from 2 to 76 signals. Event 1 dominated with a frequency of 96.45%, indicating almost complete agreement of both methods with the comparison based on individual multiple-access interference energies for ensembles of the same size. The frequency of Event 2 was 3.55%, whereas Events 3 and 4 were practically absent in this case. This result can be explained by the fact that, for ensembles of the same size, the same number of interfering signals is considered for each useful signal, and the number of signal pairs for which cross-correlations are determined is identical in both ensembles. Therefore, the difference between energy-weighted averaging and selecting the maximum individual value is minor. Under these conditions, the structural efficiency metric based on $L_{\text{eq}}^{(+)}$ showed a slight advantage when the comparison result depended on individual maximum values whose influence was not fully captured by the averaging metric.

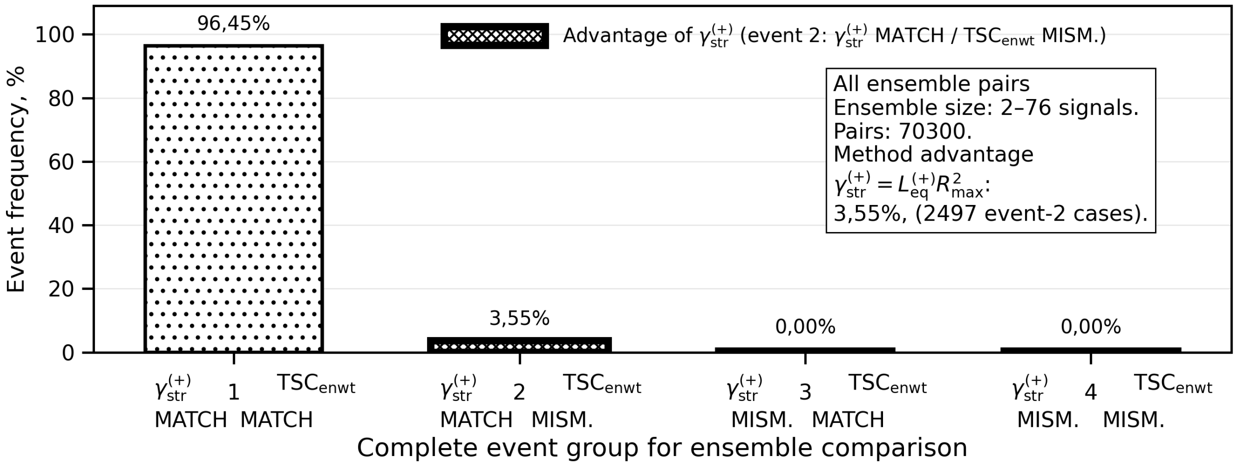

Figure 1 – Frequencies of the complete set of events when comparing the results for pairs of ensembles of the same size obtained using the structural efficiency metric $\gamma_{\text{str}}^{(+)}$ and the normalized weighted functional TSC_{enwt}, with $\gamma_{\text{str}} = LR_{max}^2$.

Figure 2 shows the event frequencies for pairs of ensembles of different sizes. Complete enumeration was performed for ensemble sizes from 2 to 76 signals. Event 2 dominated with a frequency of 72.79%, whereas Event 1 had a frequency of 26.56%, and Events 3 and 4 accounted for 0.19% and 0.46%, respectively. This distribution shows that, when comparing ensembles of different sizes, the structural efficiency metric reproduced the comparison result based on individual multiple-access interference energies considerably more often than TSC_{enwt}.

The reason is that the TSC_{enwt} functional contains an energy-weighted double sum over all signal pairs. When the ensemble size changes, the number of terms, the set of signal pairs, and the result of energy-weighted averaging also change. Therefore, the value of TSC_{enwt} may produce a comparison result different from that obtained using γ_{str}, where the maximum individual value $\gamma_{\text{str},i}$ is used for each ensemble. In contrast, the structural efficiency metric $\gamma_{\text{str}}^{(+)}$ accounts for energy nonuniformity through the structural equivalent ensemble size $L_{\text{eq}}^{(+)}$ while retaining the effect of the maximum cross-correlation R_{max}^2.

Figure 3 shows the event frequencies for the complete set of ensemble pairs, including pairs of both equal and different sizes. The event distribution was very similar to that for pairs of different sizes: Event 2 accounted for 71.59%, Event 1 for 27.77%, while Events 3 and 4 remained at 0.19% and 0.45%, respectively. Thus, the overall advantage of the structural efficiency metric was primarily observed when comparing ensembles of different sizes, because in this case the energy-weighted averaging used in TSC_{enwt} did not reproduce the comparison results based on individual multiple-access interference energies.

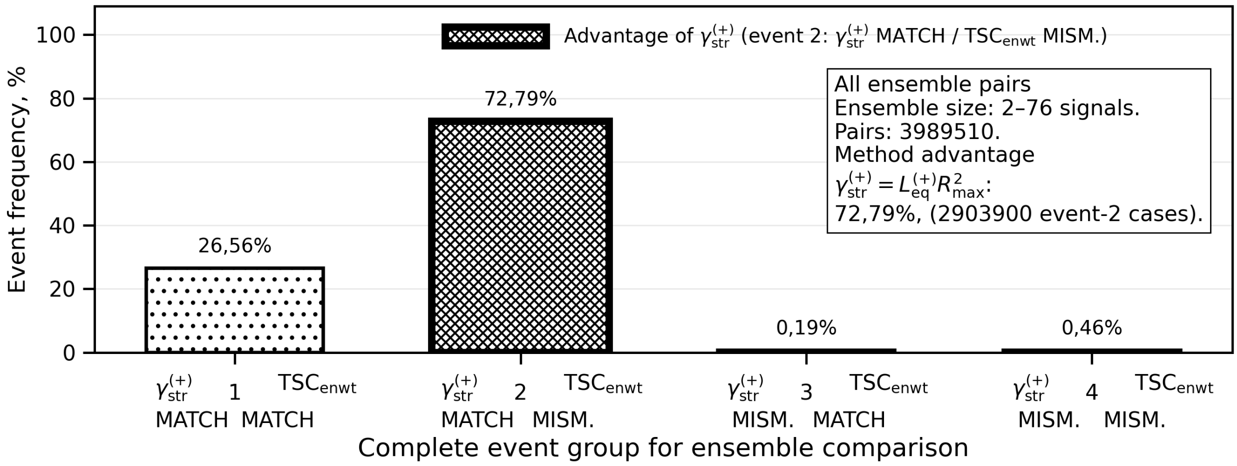

Figure 2 – Frequencies of the complete set of events when comparing the results for pairs of ensembles of different sizes obtained using the structural efficiency metric $\gamma_{str}^{(+)}$ and the normalized energy-weighted functional TSC_{enwt}, with $\gamma_{str} = LR_{max}^2$

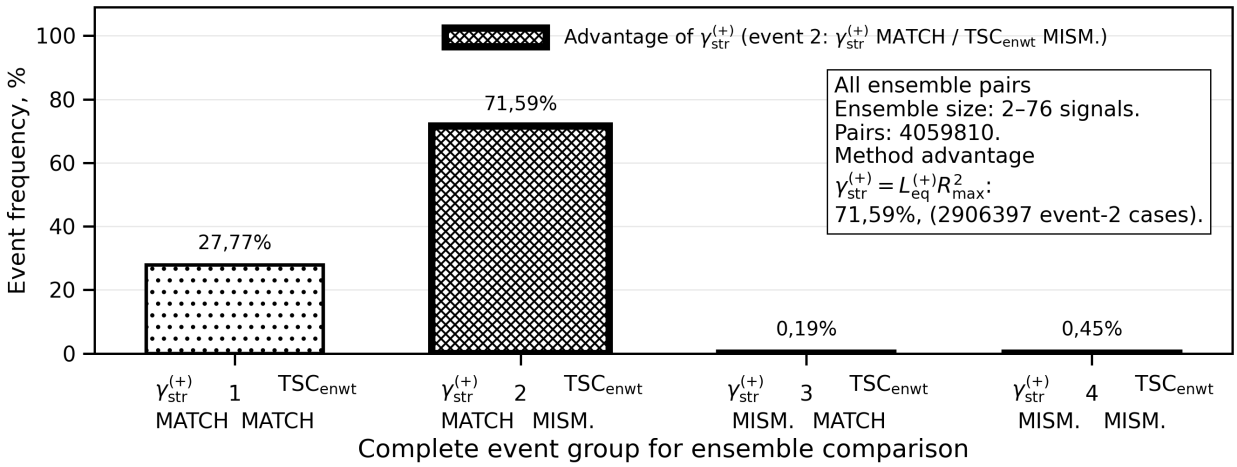

Figure 3 – Frequencies of the complete set of events when comparing the results for pairs of ensembles of both the same and different sizes obtained using the structural efficiency metric $\gamma_{str}^{(+)}$ and the normalized energy-weighted functional TSC_{enwt}, with $\gamma_{str} = LR_{max}^2$

The results presented in Figs. 1–3 confirm that the structural efficiency metric based on the structural equivalent ensemble size reproduces the results of comparing ensembles of different sizes based on individual multiple-access interference energies considerably better than the normalized energy-weighted Total Squared Correlation functional.

The computational complexity of ensemble comparison using γ_{str}, TSC_{enwt}, and $\gamma_{str}^{(+)}$ is estimated as follows. For comparison using γ_{str} , the computational complexity is $O(L^2)$ because, for each of the L signals in the ensemble, the individual multiple-access interference energy is calculated as a sum of contributions from $L-1$ interfering signals, after which the maximum value of $\gamma_{str,i}$ is selected. For comparison using the normalized energy-weighted Total Squared Correlation functional TSC_{enwt} , the computational complexity is also $O(L^2)$ because its calculation involves a complete double sum over all signal pairs and therefore requires the full set of autocorrelation and cross-correlation values, whose total number for an ensemble of size L is L^2.

For the proposed structural efficiency metric $\gamma_{str}^{(+)}$, the computational complexity is $O(L)$ because calculating $L_{eq}^{(+)}$ requires summation of the signal energies and their squares, while R_{max}^2 is treated as a previously determined or separately estimated quantity.

Thus, the computational complexity advantage of the proposed method over comparison using γ_{str} and TSC_{enwt} is:

$$\frac{O(L^2)}{O(L)} = L. \quad (28)$$

At the same time, compared with the normalized energy-weighted Total Squared Correlation functional TSC_{enwt}, the proposed structural efficiency evaluation method provides a higher agreement rate with the comparison results obtained using γ_{str}, particularly when ensembles of different sizes are compared.

Conclusions. The developed method for evaluating the structural efficiency of complex-signal ensembles uses a structural efficiency metric defined as the product of the structural equivalent ensemble size and the squared maximum cross-correlation within the ensemble. It provides agreement with the evaluation results for equal-energy complex-signal ensembles, a higher agreement rate with the results based on the maximum individual normalized multiple-access interference energy, and lower computational complexity. The computational complexity advantage of the developed algorithm over ensemble comparison based on individual normalized multiple-access interference energies or the normalized energy-weighted Total Squared Correlation functional is estimated as L, i.e., it increases linearly with the ensemble size when the maximum cross-correlation is known or estimated separately.

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