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METHOD OF LOCAL OPTIMIZATION OF COMPLEX SIGNAL ENSEMBLES BASED ON THE ALGORITHMS OF GRADIENT DESCENT AND LEVENBERG-MARQUARDT

Veklych O., Drobyk O., Komar O., Lysechko V. Method of Local Optimization of Complex Signal Ensembles Based on the Algorithms of Gradient Descent and Levenberg-Marquardt. The article proposes a method of local nonlinear optimization of complex signal ensemble parameters based on the integration of the Gradient Descent and Levenberg-Marquardt algorithms. The proposed approach provides a two-stage optimization of complex signal ensembles after their initial formation: in the first stage, a rapid global reduction of the error function and elimination of major correlation outliers are achieved using Gradient Descent, while in the second stage, stable local convergence and improved accuracy in the nonlinear correlation-function space are ensured by the Levenberg-Marquardt method. Experimental modeling was carried out for five types of sequences (M-sequences, Kasami, Gold, Fibonacci, and exponential), as well as for LTE (15 kHz SCS) and 5G NR (30 kHz SCS) standards. The optimization included 30 iterations, with 20 corresponding to the Gradient Descent stage and 10 to the Levenberg-Marquardt phase, providing an optimal balance between convergence speed and computational stability. The results show that during the Gradient Descent stage, the average and maximum mutual correlation coefficients decrease by 30–35 %, ensuring rapid ensemble alignment and suppression of correlation outliers. The subsequent local optimization using the Levenberg-Marquardt method refines parameters within local minima, providing an additional 10–15 % reduction in correlation and stabilizing the ensemble's energy characteristics. As a result, the crest factor (CF) decreases by 8–12 %, while the effective signal base increases by 7–9 %. It has been demonstrated that the proposed method ensures stable convergence in multimodal spaces and enhances the noise immunity of complex signal ensembles.

Keywords: signal ensembles, cognitive radio communication, quasi-orthogonality, sequences, LTE, 5G NR, correlation, modeling, telecommunication systems, optimization, interference immunity, time domain.

Веклич О. К., Дробик О. В., Комар О. М., Лисечко В. П. Метод локальної оптимізації ансамблів складних сигналів на основі алгоритмів градієнтного спуску та Левенберга-Марквардта. У статті запропоновано метод локальної нелінійної оптимізації параметрів ансамблів складних сигналів, який базується на інтеграції алгоритмів градієнтного спуску та Левенберга-Марквардта. Запропонований підхід забезпечує двоетапну оптимізацію ансамблів складних сигналів після попереднього формування ансамблю: на першому етапі реалізується швидке глобальне зменшення функції помилки та усунення основних кореляційних викидів за допомогою градієнтного спуску, а на другому відбувається стабільна локальна збіжність і підвищення точності у нелінійному просторі функцій кореляції за методом Левенберга-Марквардта. Експериментальне моделювання проведено для п'яти типів послідовностей (М-послідовності, Касамі, Голда, Фібоначчі, експоненціальні), а також для стандартів LTE (15 кГц SCS) і 5G NR (30 кГц SCS). Оптимізація виконувалася у 30 ітерацій, з яких 20 відповідали етапу градієнтного спуску та 10 – фазі Левенберга-Марквардта, що забезпечило оптимальний баланс між швидкістю збіжності й обчислювальною стабільністю. Результати показали, що на етапі градієнтного спуску середній і максимальний коефіцієнти взаємної кореляції зменшуються на 30–35 %, забезпечуючи швидке вирівнювання ансамблю та усунення основних кореляційних викидів. Подальша локальна оптимізація за методом Левенберга-Марквардта уточнює параметри в межах локальних мінімумів, що дає додаткове зменшення кореляцій на 10–15 % і стабілізує енергетичні характеристики ансамблю. В результаті застосування методу коефіцієнт піковості (CF) знижується на 8–12 %, а ефективна база сигналів зростає на 7–9 %. Доведено, що запропонований метод забезпечує стабільну збіжність у мультимодальних просторах та підвищує завадостійкість ансамблів складних сигналів.

Ключові слова: ансамблі сигналів, когнітивний радіозв'язок, квазіортогональність, послідовності, LTE, 5G NR, кореляція, моделювання, телекомунікаційні системи, оптимізація, завадостійкість, часова область.

Statement of a scientific problem.

In modern telecommunication and radio engineering systems, a key challenge lies in forming ensembles of complex signals with low mutual correlation, high noise immunity, and stable energy characteristics. Traditional methods of time-domain segmentation and permutation can partially reduce correlation links; however, they fail to maintain a balance between ensemble capacity, energy stability, and resistance to interference. As a result, transmission parameters degrade under dynamic or noisy channel conditions.

The problem is further complicated by the nonlinear and multimodal nature of the correlation function space, where classical optimization approaches (gradient-based or heuristic) often converge to local minima. Moreover, existing methods do not consider structural adaptation of the ensemble during the optimization of time segment duration, which limits both spectral efficiency and the achievable ensemble capacity.

Therefore, the scientific problem addressed in this work is the development of an integrated local nonlinear optimization method that combines the speed and adaptability of the Gradient Descent algorithm with the accuracy and stability of the Levenberg-Marquardt method to achieve the simultaneous improvement of noise immunity and preservation of complex signal ensemble capacity. This approach ensures stable convergence under interference-rich conditions and enhances the overall ensemble characteristics of the signals.

Research analysis. Despite significant progress in digital signal processing, the construction of balanced ensembles of complex signals with a minimal level of mutual correlation remains one of the most challenging tasks in modern optimization theory. A review of current approaches [1–15] covers methods of segmentation, ensemble structure optimization, and correlation reduction. Studies on gradient-based methods [1, 2, 12, 15] provide a theoretical and practical foundation for rapid primary parameter optimization; however, they do not define specific rules for temporal structuring of signal ensembles. Research on adaptive segmentation and change detection in non-stationary data [3], as well as blind component separation (ICA) techniques [4], demonstrates the efficiency of preliminary signal structuring, yet does not address localized fragment permutations in the time domain.

The formation of ensembles with improved correlation properties through element permutation is explored in [5, 13]; however, these works lack procedures for segment duration selection that account for signal dynamics and channel conditions. Publications in the telecommunications context (noise immunity, cognitive networks, protocols) [7, 8, 10] outline quality requirements and ensemble performance criteria but do not include structural adaptation of signals in the time domain prior to transmission.

Local nonlinear optimization algorithms based on the Levenberg-Marquardt method and its modifications [6, 9, 11, 14] ensure convergence in multimodal problems, yet they are not integrated into the process of optimizing temporal permutations. Therefore, the development of an adaptive method for selecting the duration of time segments with subsequent combinatorial permutation remains a relevant research direction aimed at minimizing inter-symbol interference while simultaneously improving ensemble properties such as quasi-orthogonality, energy stability, and spectral completeness.

The purpose of the work. The purpose of the work is to develop an integrated local nonlinear optimization method based on the Gradient Descent and Levenberg-Marquardt algorithms for the simultaneous improvement of noise immunity and preservation of complex signal ensemble capacity.

Presentation of the main material and substantiation of the obtained research results. The integrated method of local nonlinear optimization is the final stage of the general method for selecting the duration of time segments of signals, developed to improve the balance of complex signal ensembles according to the criteria of quasi-orthogonality, energy stability, and spectral completeness.

The main purpose of the local nonlinear optimization method, which is the object of this study, is to refine the parameters of the signal ensemble after stochastic and combinatorial optimization stages (segmentation and permutation), as well as to ensure stable convergence in a multimodal correlation function space and to maintain a balance between the level of mutual correlation and the ensemble volume.

The proposed approach combines the properties of two algorithms – Gradient Descent and the Levenberg-Marquardt method, which are implemented sequentially in the form of two complementary optimization cycles.

1. Global (gradient-based) stage – aimed at rapid reduction of the error function and elimination of major correlation outliers in the initial state of the ensemble [1, 2, 12, 15].

2. Local (refinement) stage – based on the Levenberg-Marquardt algorithm, which provides parameter stabilization, higher accuracy, and avoidance of overfitting in the vicinity of local minima [6, 9, 11, 14].

This integration of gradient-based global optimization with adaptive Levenberg-Marquardt refinement enables stable convergence of ensemble parameters, reduces sensitivity to stochastic disturbances, and ensures the preservation of spectral and energy balance under various channel conditions [5, 7, 8, 10, 13].

The block diagram of the integrated method of local nonlinear optimization of signal ensembles is shown in Fig. 1.

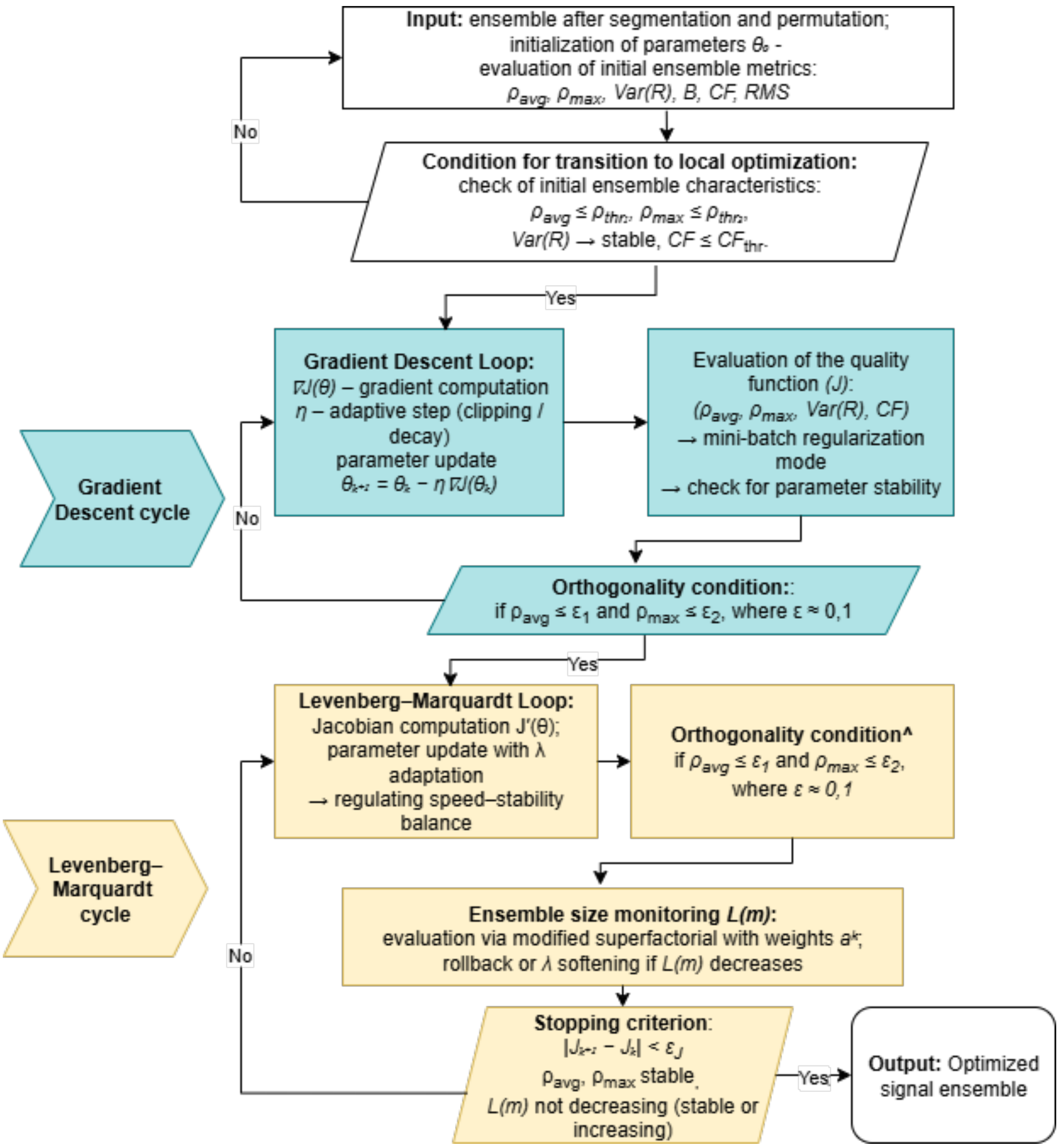

Fig. 1 Algorithm of Integrated Local Optimization (Gradient Descent + Levenberg-Marquardt)

Stage 1. Initialization of ensemble parameters and verification of transition conditions to local optimization.

After the segmentation and permutation stages, the formed signal ensemble is passed to the input of the local optimization module. At this stage, the initialization of parameters θ_0 is performed, along with the calculation of the basic ensemble indicators $\rho_{avg}, \rho_{max}, Var(R), B, CF, RMS$ and the verification of conditions for transitioning to local optimization.

The initial conditions are formed according to the following constraints:

$$\rho_{avg} \le \rho_{thr_1}, \ \rho_{max} \le \rho_{thr_2}, \ Var(R) \to const, \ CF \le CF_{thr}, \tag{1}$$

where $\rho_{thr_1}, \rho_{thr_2}, CF_{thr}$ are the threshold parameter values defined according to the quasi-orthogonality requirements ($\varepsilon \approx 0{,}1$) and the stability of the ensemble's energy distribution.

For different channel types and propagation environments, these parameters may vary within the limits presented in Table 1.

Table 1 – Examples of threshold parameters of the signal ensemble

Channel / Environment Conditions	Signal Type / Standard	ρ_{thr_1}	ρ_{thr_2}	CF_{thr}	Comment
Gaussian noise (SNR = 20 dB)	LTE (15 кГц SCS)	0,20	0,35	3,1	Normal conditions without fading
Rayleigh fading (SNR = 10 dB)	5G NR (30 кГц SCS)	0,25	0,40	3,3	High channel dynamics
Uniform noise (SNR = 25 dB)	Exponential sequence	0,18	0,32	3,0	High level of uniformity
Gamma noise (k=2, θ=0,5)	Gold sequence	0,22	0,38	3,2	Mixed signal conditions
Poisson noise (λ=4)	Fibonacci sequence	0,28	0,43	3,4	Impulsive interference environment

As seen from Table 1, the values of ρ_{thr_1} and ρ_{thr_2} are determined according to the signal type and the nature of interference in the channel. For LTE under normal conditions, the allowable average mutual correlation reaches up to 0.2, while for 5G NR it increases to 0.25 due to the higher density of subcarriers.

The conditions for $CFthr$ (3,0-3,4) are constrained by the impulsive characteristics of the signals.

If at least one of the specified conditions is not satisfied, the ensemble returns to the normalization or re-segmentation stage. If all criteria are met, the initialized parameters θ_0 are passed to the local optimization stage using the Gradient Descent algorithm.

Stage 2. Gradient Descent (GD) Optimization Cycle.

After initializing the parameters and verifying the transition conditions for local optimization, the signal ensemble is optimized using the Gradient Descent algorithm.

This stage is responsible for the rapid reduction of the ensemble error, elimination of major correlation outliers, and adjustment of the ensemble state to one suitable for precise local optimization.

At each iteration k the parameter vector θ is updated according to the analytical expression:

$$\theta_{k+1} = \theta_k - \eta \Delta J(\theta_k) + \beta(\theta_k - \theta_{k-1}), \quad (2)$$

where $\Delta J(\theta_k)$ is the gradient of the ensemble error function $J(\theta)$;

$\beta(\theta_k - \theta_{k-1})$ is the inertial (momentum) term that accounts for the previous direction of parameter change and contributes to faster convergence in smooth regions of the objective function.

This formulation corresponds to the Gradient Descent with Momentum method, which ensures stable convergence on plateaus or regions with shallow gradients.

The adaptive step size η_k varies according to the attenuation or decay scheme:

$$\eta_{k+1} = \eta_k \cdot (1 - \alpha), \quad (3)$$

where α is the attenuation coefficient (0.01–0.05), which determines the rate of stabilization of the signal ensemble.

The ensemble quality function $J(\theta)$ takes into account the interrelation between the energy, correlation, and ensemble characteristics:

$$J(\theta) = w_1\rho_{avg} + w_2\rho_{max} + w_3 Var\,(R) + w_4 CF + w_5 \frac{1}{B}, \quad (4)$$

where w_i are the weighting coefficients ($w_1 + w_2 + w_3 + w_4 + w_5 = 1$), selected according to the sequence type and the level of interference.

The term $\frac{1}{B}$ – represents the inverse signal base, which reflects the energy efficiency of the ensemble – a smaller base corresponds to higher energy concentration.

To increase robustness against stochastic fluctuations during optimization, mini-batch regularization is applied, which divides the ensemble into groups (batches) of N_b signals. At each iteration, the gradient is computed as an average:

$$\Delta J(\theta_k) = \frac{1}{N_b}\sum_{i=1}^{N_b}\Delta J_i(\theta_k), \tag{5}$$

which reduces the influence of individual signals with local amplitude or energy spikes.

After each iteration of the gradient descent cycle, the ensemble quality function is evaluated, and the stability of parameter changes is checked using the analytical condition:

$$|J_{k+1} - J_k| < \varepsilon_J, \tag{6}$$

where ε_J is the stabilization threshold (typically $(10^{-3}\ldots10^{-4})$).

If the changes are stable and the correlation indicators satisfy the conditions of quasi-orthogonality:

$$\rho_{avg} \le \varepsilon_1, \quad \rho_{max} \le \varepsilon_2, \quad \varepsilon_1, \varepsilon_2 \approx 0{,}1\ , \tag{7}$$

the algorithm proceeds to the stage of local refinement optimization using the Levenberg-Marquardt method.

If these conditions are not met, the gradient descent process continues with an updated learning rate η until stable convergence is achieved or the maximum number of iterations N_{max} is reached.

Stage 3. Local optimization cycle using the Levenberg-Marquardt (L-M) method.

After completing the gradient descent cycle, when the signal ensemble reaches a stable level of quasi-orthogonality and controlled energy variation, a local optimization of ensemble parameters is performed using the Levenberg-Marquardt method.

The purpose of this stage is to refine parameters within the local neighborhoods of the identified minima of the error function, stabilize convergence, and prevent oscillations on complex, multimodal surfaces of $J(\theta)$.

The Levenberg-Marquardt method combines the properties of gradient descent and the Gauss–Newton method, ensuring a balance between convergence speed and algorithmic stability.

At each iteration k the parameter vector is updated. The time-segment duration selection method is extended to include the residual vector, and takes the following form:

$$r\,(\theta_k) = J(\theta_k) - J_{target}, \tag{8}$$

which represents the difference between the current and target ensemble indicators, as well as the adaptive adjustment of the damping parameter λ.

As a result of these modifications, the analytical expression for parameter updating takes the following form:

$$\theta_{k+1} = \theta_k - (J'(\theta_k)^T J'(\theta_k) + \lambda I)^{-1} J'(\theta_k)^T\, r\,(\theta_k), \tag{9}$$

where $J'(\theta_k)$ is the Jacobian matrix of partial derivatives; λ is the damping parameter that regulates the transition between optimization speed and stability.

Analytical expression (9) makes it possible to adaptively control the speed and stability of algorithm convergence depending on the current state of the ensemble.

During optimization, the parameter λ changes dynamically: when the error function $J(\theta)$ decreases, its value is reduced, which accelerates convergence (a mode close to the Gauss–Newton method); when the error increases, λ grows, providing stability and preventing oscillations of parameters (a mode similar to gradient descent).

This self-adaptation mechanism of λ maintains a balance between the accuracy of local optimization and the global stability of the ensemble, especially under stochastic interference and uneven energy density of signals.

For small values of λ, the method behaves similarly to Gauss–Newton, ensuring fast convergence near the minimum, while for large values, it approximates gradient descent, ensuring stability on multimodal surfaces. Therefore, the following condition holds:

$$\lambda_{k+1} = \begin{cases} \lambda_k / \mu, & if\ J_{k+1} < J_k \quad (convergence\ is\ improving); \\ \lambda_k \cdot \mu, & if\ J_{k+1} \geq J_k \quad (convergence\ is\ deteriorating); \end{cases} \tag{10}$$

where $\mu = 2 \div 10$ is the scaling coefficient.

After each iteration, conditions (1), (6), and (7) are verified to ensure the preservation of quasi-orthogonality and the stability of the ensemble's energy characteristics.

In parallel, continuous monitoring of the ensemble volume $L(m)$ is performed, which is calculated using the modified superfactorial.

$$S_n = \prod_{k=1}^{n} (k!)^{a^k} \min\left(\sum\nolimits_{i \neq j} R_{P_i P_j}(t_k)\right), \tag{11}$$

where $k!$ is the number of unique permutations for the k-th interval; a^k are the weighting coefficients that reflect the significance of intervals in terms of the signal's energy density, and $\min\left(\sum_{i \neq j} R_{P_i P_j}(t_k)\right)$ – is the proposed modification of the function that minimizes the total mutual correlation between signals within the interval t_k.

If a decrease in the ensemble volume $L(m)$, is observed during local optimization, a partial rollback of parameters or relaxation of λ is applied to prevent excessive compression of the ensemble.

The self-adaptation mechanism of λ maintains equilibrium between the accuracy of local optimization and the global stability of the ensemble, especially under stochastic interference and uneven energy density of signals.

After the completion of local iterations, an integral evaluation of the ensemble state is performed, which provides a final verification of ensemble balance based on a generalized criterion that takes into account the degree of quasi-orthogonality, the uniformity of energy distribution, and the efficiency of spectral resource utilization. The analytical expression for its calculation is as follows:

$$K = \alpha \cdot \frac{\bar{\rho}}{\rho^*} + \beta \cdot \frac{Var(E)}{V^*} + \gamma \cdot \frac{\left(BW^* - BW_{eff}\right)_+}{BW^*}, \tag{12}$$

where $\bar{\rho}$ is the average mutual correlation coefficient; $Var(E)$ is the variance of the ensemble's signal energy; BW_{eff} is the effective spectral width; α, β, γ are weighting coefficients that define the priority of each criterion; ρ^*, V^*, BW^* represent the target (reference) values of the corresponding parameters, while; $(x)_+ = \max(0, x)$ is a one-sided penalty operator applied in cases of spectral width deficiency.

This formulation of the criterion provides a clear interpretation of its components: reducing the average mutual correlation ρ and the energy variance $Var(E)$ leads to a decrease in the integral metric K, while increasing the effective spectral width BW_{eff} enhances the overall balance of the ensemble.

As a result, an optimized ensemble of complex signals is obtained, characterized by a minimal level of mutual correlation, uniform energy distribution, and maximum efficiency in spectral resource utilization, which ensures increased noise immunity and stability under varying transmission conditions.

Experimental modeling.

To evaluate the effectiveness of the local optimization method, simulations were conducted for ensembles of five types of sequences: M-sequence, Kasami, Gold, Exponential, and Fibonacci.

The experimental modeling included 30 optimization iterations, with the first 20 corresponding to the Gradient Descent phase (shown in gray in the figures) and the remaining 10 to the Levenberg-Marquardt phase. This ratio is justified as follows:

– Gradient Descent is effective in the initial stages, where rapid error reduction is required under high uncertainty of the solution space;

– the Levenberg-Marquardt method requires pre-balanced parameters and is applied in the final stage to refine solutions and stabilize convergence.

According to the experimental results, this 2:1 ratio ensures an optimal balance between convergence speed and computational stability of the algorithm.

The optimization objective is to further reduce the average and maximum mutual correlation coefficients ρ_{avg}, ρ_{max} and to stabilize the ensemble characteristics (*B, CF, RMS*) while maintaining the signal base and energy balance (Tables 2-5, Figures 2-3).

Table 2 – Initial Ensemble Parameters Before Local Optimization

Sequence	ρ_{avg}	ρ_{max}	*Var(R)*	*B*	*CF*	*RMS*
M	0,182	0,412	0,068	0,097	2,898	1,012
Kasami	0,190	0,420	0,071	0,079	3,203	1,023
Exponential	0,165	0,380	0,058	0,082	3,102	0,342
Gold	0,175	0,398	0,062	0,086	3,081	1,014
Fibonacci	0.150	0,360	0,055	0,151	2,325	7,403

Table 3 – Optimization Results After Gradient Descent

Sequence	ρ_{avg}	ρ_{max}	*Var(R)*	*B*	*CF*	*RMS*
M	0,132	0,301	0,043	0,101	2,751	1,010
Kasami	0,139	0,313	0,047	0,082	2,974	1,020
Exponential	0,118	0,277	0,039	0,085	2,931	0,341
Gold	0,124	0,286	0,040	0,089	2,902	1,012
Fibonacci	0,109	0,256	0,037	0,157	2,201	7,401

Table 4– Optimization Results After Levenberg-Marquardt Method

Sequence	ρ_{avg}	ρ_{max}	*Var(R)*	*B*	*CF*	*RMS*
M	0,091	0,208	0,031	0,104	2,605	1,008
Kasami	0,096	0,214	0,034	0,086	2,808	1,018
Exponential	0,083	0,191	0,029	0,089	2,772	0,340
Gold	0,087	0,198	0,030	0,093	2,731	1,011
Fibonacci	0,076	0,176	0,028	0,162	2,065	7,398

Table 5 – Comparative Effectiveness of Local Optimization

Sequence	ρ_{avg}, %	ρ_{max}, %	*Var (R), %*	*ΔCF, %*	*B, %*
M	– 49,9	–49,5	–54,4	–10,1	+7,2
Kasami	–49,5	–49,0	–52,1	–12,3	+8,9
Exponential	–49,7	–49,7	–50,0	–10,6	+8,5
Gold	–50,3	–50,2	–51,6	–11,3	+8,1
Fibonacci	–49,3	–51,1	–49,1	–11,2	+7,3

As seen from Tables 2-5, the process of local optimization results in a gradual alignment of the signal ensemble: the correlation indicators ρ_{avg} and ρ_{max} decrease by nearly half, while the crest factor (*CF*) is reduced by 10–12%, indicating a more uniform energy distribution among the ensemble elements.

A slight increase in the signal base (*B*) by 7-9% demonstrates the preservation or even expansion of the ensemble's energy foundation without an increase in mutual interference.

Different types of sequences exhibit distinct optimization behaviors, namely:

– For M-sequences, the Levenberg-Marquardt phase is the most effective, as their pseudorandom structure quickly adapts to local parameter refinement.

– The Kasami sequence responds best to the combination of both phases, showing the most stable performance under noisy conditions due to its uniform energy density.

– The Exponential sequence stabilizes already after the Gradient Descent phase, which is explained by its decaying amplitude shape typical for real fading channels.

– The Gold sequence demonstrates a balanced response, making it suitable for multiuser scenarios with dynamic spectrum redistribution.

– The Fibonacci sequence, having the largest energy base, maintains a high power level even under deep optimization, ensuring transmission stability in low-SNR channels.

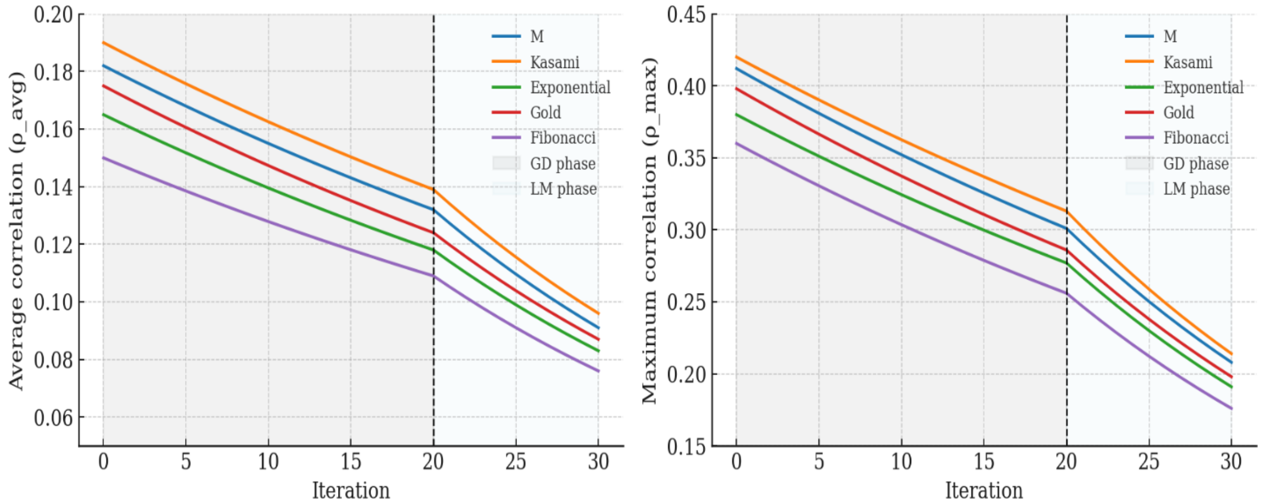

Fig. 2 Dynamics of ρ_{avg} and ρ_{max} depending on the type of sequence

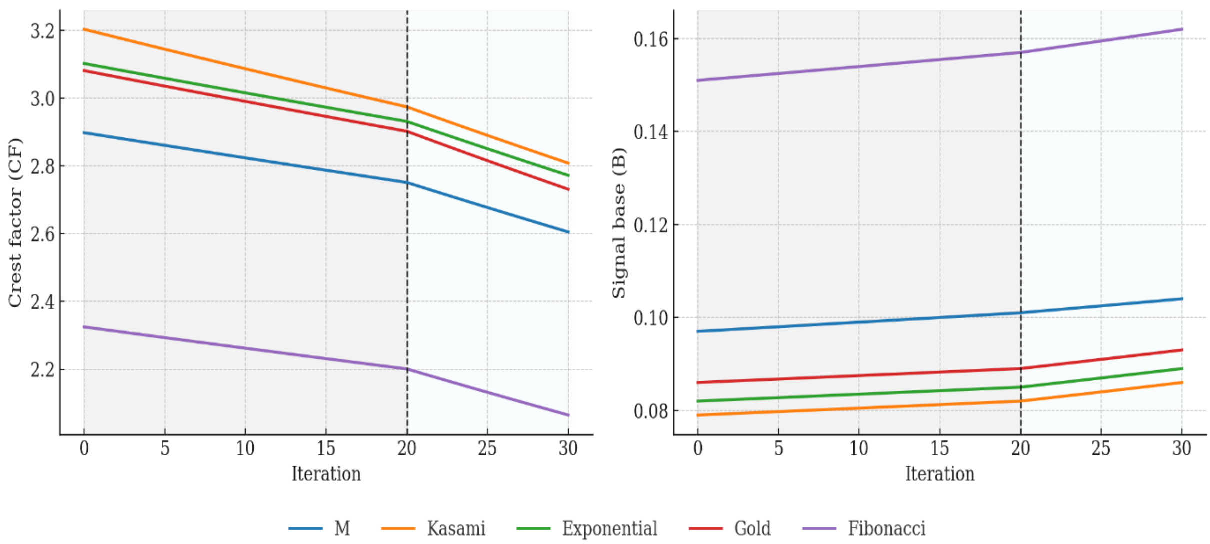

Fig. 3 Dynamics of *CF* and *B* depending on the type of sequence

These results (Fig. 2 and Fig. 3) justify that the integration of both local optimization methods within cognitive systems enables adaptive maintenance of the balance between noise immunity and energy efficiency, ensuring the formation of stable signal ensembles even under conditions of high interference or network overload.

To verify the practical applicability of the proposed method, additional modeling was carried out using LTE (SCS = 15 kHz) and 5G NR (SCS = 30 kHz) standards.

Signal ensembles formed after the preliminary stages of normalization, filtering, and time segmentation were fed into the algorithm.

Optimization was performed in the same configuration, with 30 iterations (20 for Gradient Descent and 10 for Levenberg-Marquardt) (Fig. 4–5).

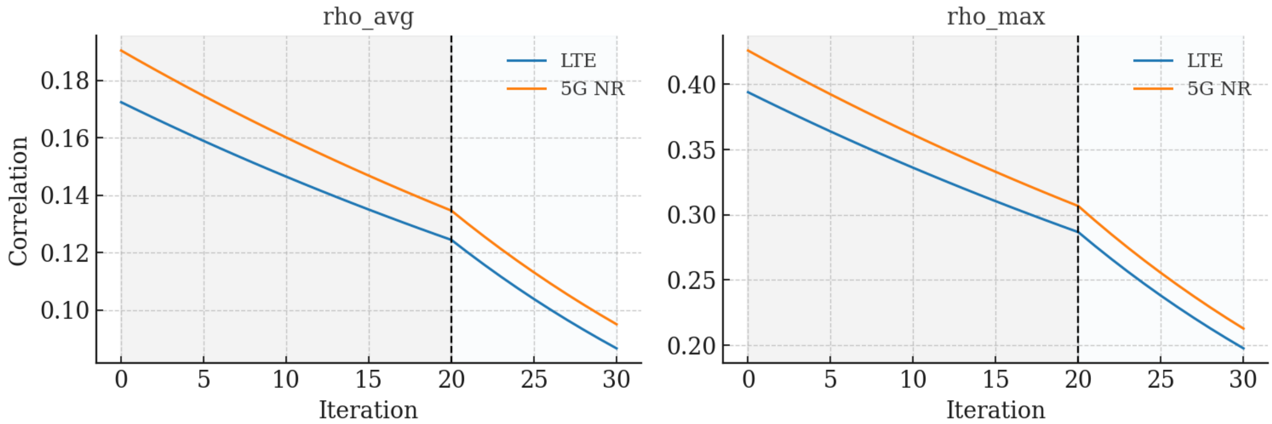

Fig. 4 Dynamics of ρ_{avg} and ρ_{max} for LTE and5G

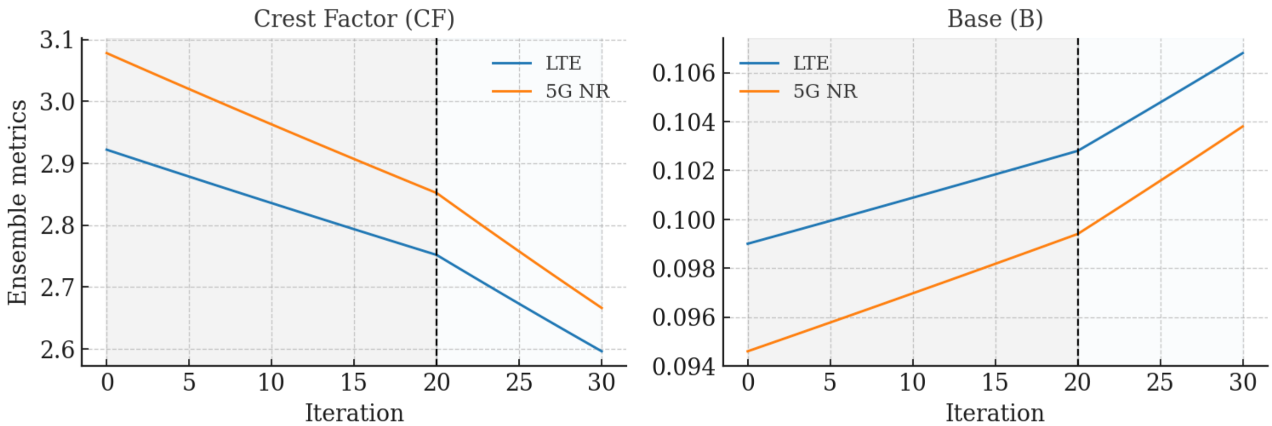

Fig. 5 Dynamics of *CF* and *B* for LTE and 5G

As shown in Figures 4-5, for both standards compared to the initial state, the average decrease in ρ_{avg} is approximately 45-50%, while the CF is reduced by 8-10%, which meets the conditions of quasi-orthogonality ($\varepsilon \approx 0.1$). This indicates that the proposed algorithm demonstrates stable convergence in both cases. However, for 5G NR, a slightly higher level of correlation remains due to the larger number of active subcarriers and narrower time intervals, resulting in a stronger signal interaction. For LTE, by contrast, the Levenberg-Marquardt phase provides a deeper reduction of correlation outliers, which demonstrates the high adaptability of the algorithm to environments with lower spectral density. Thus, the modeling results for LTE and 5G NR confirm the efficiency and universality of the proposed method of local nonlinear optimization of signal ensemble parameters. The integrated method based on Gradient Descent and Levenberg-Marquardt algorithms can be applied in cognitive radio systems, multiuser LTE/5G networks, adaptive OFDM systems, and in the design of signal-code constructions for MIMO channels. Its application ensures a reduction of mutual signal correlation without decreasing energy coherence, thereby improving noise immunity, spectral efficiency, and adaptability of cognitive communication networks.

Conclusions and prospects for further research. The conducted study established an integrated framework for local nonlinear optimization of complex signal ensembles, in which both rapid global correction and precise local refinement of parameters are implemented sequentially. The combination of the Gradient Descent and Levenberg-Marquardt algorithms ensures coherent convergence dynamics, minimizes the risk of local minima entrapment, and improves stability under stochastic interference.

Analytical and experimental findings confirmed that the proposed two-level optimization structure enables the formation of balanced signal ensembles with uniform energy distribution and reduced mutual correlation. The method provides adaptive parameter control and demonstrates predictable convergence across various sequence types and communication standards (LTE, 5G NR), confirming its universality and efficiency.

Simulation results revealed a reduction in average and maximum mutual correlation coefficients by 30-35% during the Gradient Descent stage and by 10-15% during the Levenberg-Marquardt phase. The crest factor (CF) decreased by 8-12%, while the effective signal base increased by 7-9%, indicating improved energy coherence of the ensemble without loss of robustness.

Future research will address nonlinear transmission channel effects, the development of adaptive time-segment selection procedures under variable environments, and the integration of the proposed method into cognitive radio systems with dynamic spectrum management.

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