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APPLICATION OF A CYCLIC STOCHASTIC PROCESS MODEL FOR ANALYZING ELECTRICITY CONSUMPTION

Voloshchuk A., Osukhivska H. Application of a Cyclic Stochastic Process Model for Analyzing Electricity Consumption. The accuracy of electricity consumption forecasting is a critical factor for the efficiency of modern power systems; however, traditional approaches rely on the assumption of deterministic seasonality, neglecting the stochastic variability of cyclic parameters. The objective of this study is to validate a mathematical model of electricity consumption as a cyclical random process to adequately describe the stochastic nature of cyclic fluctuations. The paper employs the mathematical framework of cyclical random processes and conducts a statistical analysis of real-world hourly electricity consumption data acquired from residential smart meters. A model is proposed that enables the structural modeling of the randomness inherent in the cycle parameters themselves (amplitude and phase), in contrast to existing approaches that only model fluctuations around a fixed cycle. Empirical analysis confirmed periodic variations in mathematical expectation, variance, and higher-order moments, thereby demonstrating the cyclical nature of the process and the inadequacy of traditional stationary models. The correlation analysis revealed distinctive peaks in the autocovariance function, indicating the system's "memory" of its diurnal cycle. The results hold practical significance for enhancing forecasting accuracy in power systems and can be adapted for the analysis of other cyclic processes across various domains.
Keywords: electricity consumption, model, cyclical random process, statistical characteristics, mathematical modeling

Волощук А. В., Осухівська Г. М. Застосування моделі циклічного стохастичного процесу для аналізу електроспоживання.Точність прогнозування електроспоживання є критичним фактором ефективності сучасних енергосистем, проте традиційні підходи базуються на припущенні детермінованої сезонності, не враховуючи стохастичну мінливість циклічних параметрів. Метою дослідження є валідація математичної моделі електроспоживання як циклічний випадкового процесу для адекватного опису стохастичної природи циклічних коливань. У роботі застосовано математичний апарат циклічних випадкових процесів та проведено статистичний аналіз реальних даних погодинного електроспоживання приватного господарства, отриманих від інтелектуальних лічильників. Запропоновано модель, яка дозволяє структурно моделювати випадковість параметрів самого циклу (амплітуди та фази), на відміну від існуючих підходів, що моделюють лише флуктуації навколо фіксованого циклу. Емпіричний аналіз підтвердив періодичні зміни математичного сподівання, дисперсії та моментів вищих порядків, що доводить циклічну природу процесу та неадекватність традиційних стаціонарних моделей. Кореляційний аналіз виявив характерні піки автоковаріаційної функції, що свідчать про "пам'ять" системи щодо добового циклу. Результати мають практичне значення для підвищення точності прогнозування в енергосистемах та можуть бути адаптовані для аналізу інших циклічних процесів у різних галузях.
Ключові слова: електроспоживання, модель, циклічний випадковий процес, статистичні характеристики, математичне моделювання

Problem statement. Stable functioning of energy systems is vital for economic development, industrial productivity, and social welfare. Simultaneously, management in the electrical energy sector today is impossible without the application of innovative information technologies capable of ensuring high efficiency in planning, monitoring, and forecasting electricity consumption. Modern energy is undergoing active transformation driven by economic dependence on electrical energy, the development of digital control technologies, and widespread implementation of renewable energy sources (RES) [1, 2]. These processes form a new concept of energy system operation flexible, decentralized, but simultaneously more complex and sensitive to uncertainties. The issue of balancing electricity production and consumption becomes particularly relevant, as the stochastic nature of RES generation and behavioral changes in consumers lead to increased load irregularities in the energy system [3, 4].

Under the development of the Smart Grid concept [5, 6] and active implementation of Big Data-based control systems, the accuracy of electricity consumption forecasting becomes a determining factor for the efficiency of energy markets and reliability of system planning. Forecasting errors can lead to significant economic losses, network overloads, system stability degradation, and irrational use of power reserves. Therefore, improving forecasting accuracy is one of the key tasks of modern energy analytics [7].

Despite significant progress in applying classical statistical methods and machine learning algorithms, most existing approaches are based on the assumption of deterministic seasonality, treating daily or weekly cycles as fixed periodic components. Such an approach does not account for stochastic variability changes in amplitude and phase caused by behavioral, climatic, or technological factors. As a result, model accuracy decreases precisely where it is most needed when forecasting consumption of individual residential or industrial facilities, where stochasticity manifests most strongly.

In view of this, the objective of the study is to model residential electricity consumption as a cyclicality random process model capable of adequately describing the stochastic nature of cyclical fluctuations in electricity consumption, which will enable the use of obtained stochastic estimates in analyzing the electricity consumption process. The scientific novelty of the work lies in investigating a model with dual stochasticity, which, unlike existing approaches (that model only fluctuations around the cycle), allows for structural modeling of the randomness of the cycle parameters themselves its amplitude and phase.

Analysis of the latest research and publications. Electricity consumption analysis is an important research area that has been actively pursued in recent years. Researchers have developed various approaches that can be divided into four main categories: statistical methods, machine learning methods, decomposition-combination techniques, and hybrid models [8]. Statistical methods, represented by the SARIMA model [10-12], are based on rigorous mathematical description of time series. Their advantages include high interpretability and theoretical foundation. However, their limitation is the assumption of deterministic seasonality. They work effectively when cycles are stable and regular, but lose accuracy when the shape, amplitude, or duration of cycles undergo random fluctuations, which is characteristic of real data.

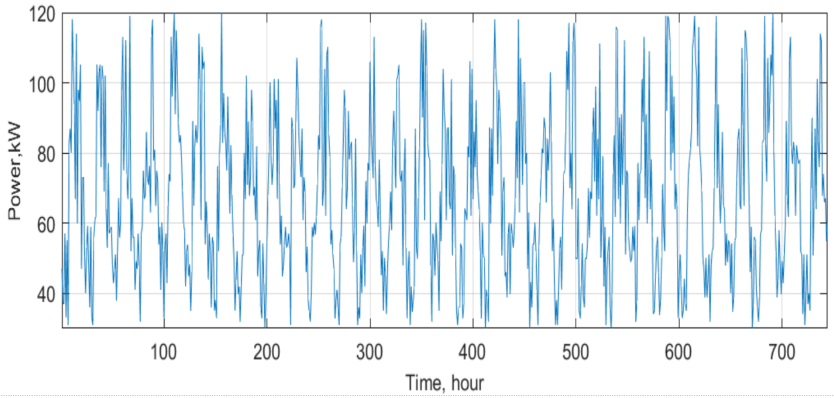
The accuracy of electricity consumption forecasting depends on numerous factors that reflect nonlinear characteristics in the data, making machine learning and artificial neural networks indispensable components of modern load forecasting [9]. Machine learning and deep learning methods for electricity consumption analysis, such as recurrent neural networks (LSTM, GRU) [13, 14, 15] and attention mechanism-based architectures (Transformers) [16], demonstrate high efficiency in capturing complex nonlinear dependencies without explicit modeling of process structure. However, they are not entirely suitable for critical systems due to low interpretability (black box principle), which prevents deep understanding of their decision logic and complicates diagnosis of error causes. Although they are capable of learning complex patterns, they do not provide a structural model of the stochasticity of the cycle itself. A "black box" model (such as LSTM or Transformer) learns from the consequences of this variability. It sees that the overall pattern is different each day and attempts to find a complex nonlinear function to account for this, but does so implicitly.

Decomposition-combination and hybrid approaches [17, 18] are attempts to overcome the limitations of individual methods. Decomposition techniques (for example, based on wavelet transform or empirical mode decomposition) first separate the time series into simpler components (trend, seasonality, residuals), forecasting each separately. Hybrid models combine the strengths of different classes, for example, using SARIMA for the linear part and neural networks for modeling nonlinear residuals [19]. However, despite improved accuracy, the main cyclicality is still considered as a deterministic structure that needs to be "cleaned" of noise, rather than as a dynamic process with stochastic parameters that needs to be modeled. Recent developments include the use of graph attention networks to account for spatio-temporal features of energy systems.

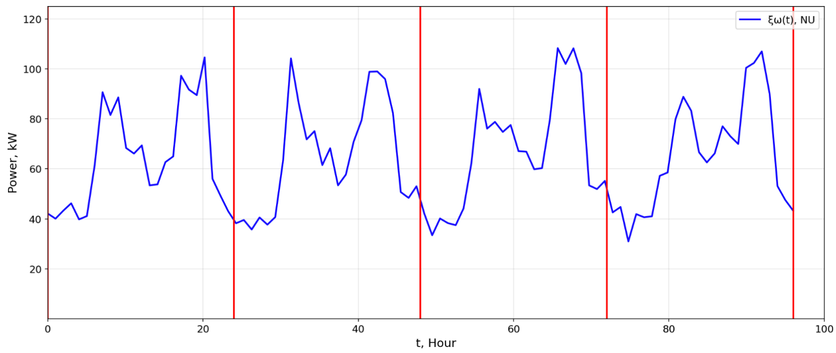
Thus, comprehensive analysis of existing approaches (statistical, ML, decomposition, and hybrid) reveals a scientific problem: the need for a model that would combine the mathematical rigor and interpretability of statistical methods with the ability to adequately model the stochasticity of the cyclical structure itself, rather than just fluctuations around it. Therefore, there is a clear need to transition from the "deterministic seasonality and noise" paradigm to a model that describes random dynamics of cycle parameters (amplitude, phase, shape). The necessity of such an approach is confirmed by works [20], which demonstrate the effectiveness of applying stochastic models for analyzing electricity consumption processes.

Precisely to solve this problem, it was proposed to depart from classical models and consider the electricity consumption process as a periodically correlated (or cyclicality) random process. This approach allows adequate description of processes whose statistical characteristics change periodically over time, which fully corresponds to the nature of diurnal cycles. Specifically, work [21] applied a component analysis method for signal decomposition. This work proposes representing the electricity consumption signal as a cyclicality random process, which will allow using the obtained stochastic estimates in modeling the electricity consumption process in further research to select the most adequate model.

Presenting the main material. For processing electricity consumption signals, we propose the use of a mathematical model based on cyclicality random processes. This model allows for adequate description of the stochastic nature of cyclical fluctuations in electricity consumption, in contrast to classical approaches that treat seasonality as a deterministic component. In this study, we analyze time series data reflecting hourly electricity consumption of a residential household, collected over several months using smart meters. The sample is sufficient for conducting detailed analysis of the statistical structure and empirical verification of the properties of the proposed model. The electricity consumption data is proposed to be represented as a second-order cyclicality random process [22, 23] $\xi_\omega(t)$ with a fundamental period T=24 hours. Cyclicality means that the statistical characteristics of the process change periodically over time, which fully corresponds to the nature of diurnal cycles of electricity consumption (Figure 1).



a)



b)

Fig. 1. Residential household electricity consumption data:
a) electricity consumption over a month; b) electricity consumption over four complete diurnal cycles.

As can be seen from Figure 1, the process exhibits a clearly pronounced cyclical character, however, the shape and amplitude of each cycle vary. Such stochastic variability provides visual confirmation of the validity of the proposed model. For quantitative analysis of the model properties, a study of second-order moment functions was conducted.

For further analysis of the process properties, we proceed to examine its probabilistic characteristics averaged within one cycle.

The implementation of statistical estimation of the average electricity consumption level $m_{\xi_\omega}(t)$ is calculated by averaging over all M cycles (one day) with period T:

$$\widehat{m}_{\xi_i}(t)=\frac{1}{M}\sum_{n=0}^{M-1}\xi_{i_\omega}\left(t+T(t,n)\right), i=\overline{1,N}$$

This relationship means that the averaged profile corresponds to the baseline schedule. Figure 2 presents the realization of the statistical estimation of the mathematical expectation $m_{\xi_\omega}(t)$, which reflects the averaged daily electricity consumption profile and corresponds to the component $m(t)$.

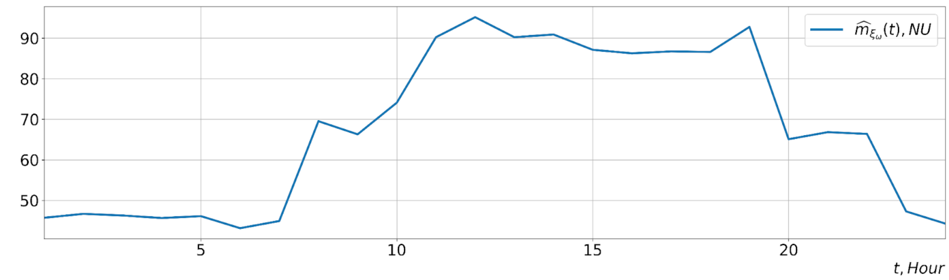


Fig. 2. Graph of realizations of statistical estimates of mathematical expectation of electricity consumption signal of a residential household.

The graph (Figure 2) demonstrates the average daily electricity consumption profile of a residential household. The mathematical expectation is not a constant, which confirms the non-stationarity of the studied time series. The next stage of analysis is the investigation of variance $D_\xi(t)$, which characterizes the power of random fluctuations or, in other words, the level of uncertainty of the process at each moment. The realization of statistical estimation of variance $\widehat{m}_\xi(t)$ of each component of electrical load $\xi_\omega(t)$, which characterizes the variability of electricity consumption.

$$\hat{d}_{\xi_i}(t)=\frac{1}{M}\cdot\sum_{n=0}^{M-1}\left[\xi_{i_\omega}\left(t+T(t,n)\right)-\widehat{m}_{\xi_i}\left(t+T(t,n)\right)\right]^2, i=\overline{1,N}$$

Figure 3 demonstrates the graphs of realizations of statistical estimates of variance of the electricity consumption signal.

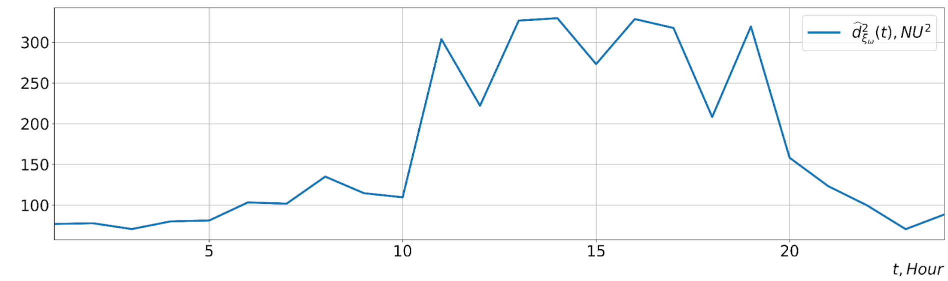


Fig. 3. Graphs of realizations of statistical estimates of variance of electricity consumption signal.

The variance is not constant; it takes minimum values at night when consumption is stable and predictable, and reaches maximum during peak load hours (see Fig. 3), which indicates the greatest variability of consumption during this period.

For a more complete description of the univariate distributions of the process, initial moments of higher orders were analyzed, particularly statistical estimates of the initial moment of second and third order (Figure 4 and Figure 5). Application of statistical estimation for the function of the initial moment of k-th order $\widehat{m}_{\xi_i}{}^k(t)$ of each element $\xi_i(\omega,t)$.

$$\widehat{m}_{\xi_i}(t)=\frac{1}{M}\cdot\sum_{n=0}^{M-1}\left[\xi_{i_\omega}\left(t+T(t,n)\right)-\widehat{m}_{\xi_i}\left(t+T(t,n)\right)\right]^2, i=\overline{1,N}$$

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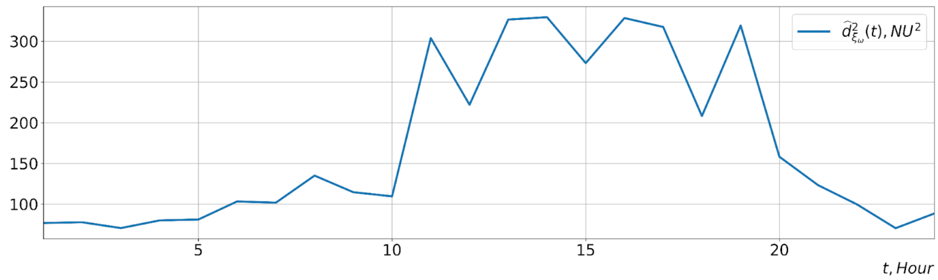

Fig. 4. Graphs of realizations of statistical estimates of variance of electricity consumption signal.

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For a more complete description of the univariate distributions of the process, initial moments of higher orders were analyzed, particularly statistical estimates of the initial moment of second and third order (Figure 4 and Figure 5). Application of statistical estimation for the function of the initial moment of k-th order $\widehat{m}_{\xi_i}{}^k(t)$ of each element $\xi_i(\omega, t)$.

$$\widehat{m}_{\xi_i}{}^k(t) = \frac{1}{M} \sum_{n=0}^{M-1} \xi_{i_\omega}^k \left(t + T(t,n)\right), i = \overline{1, N}$$

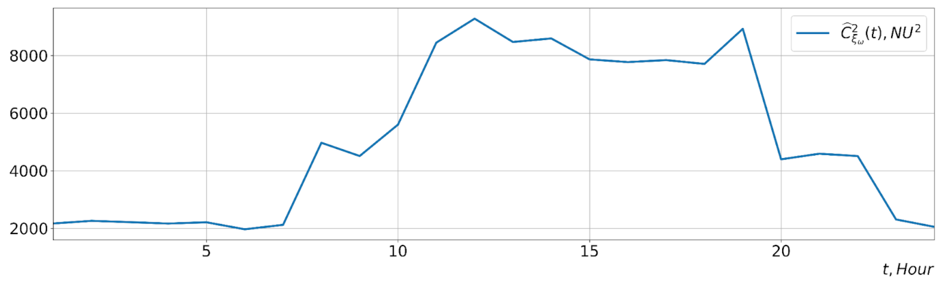

Fig.5. Graph of realization of statistical estimation of the second-order central moment function $\hat{C}^2_{\xi_\omega}(t)$.

Calculation of empirical estimation of k-th central moment $\hat{d}_{\xi_i}{}^k(t)$ of each component of the stochastic process $\xi_i(\omega, t)$

$$\hat{d}_{\xi_i}{}^k(t) = \frac{1}{M-1} \cdot \sum_{n=0}^{M-1} \left[\xi_{i_\omega}\left(t + T(t,n)\right) - \widehat{m}_{\xi_i}\left(t + T(t,n)\right)\right]^k, i = \overline{1, N}$$

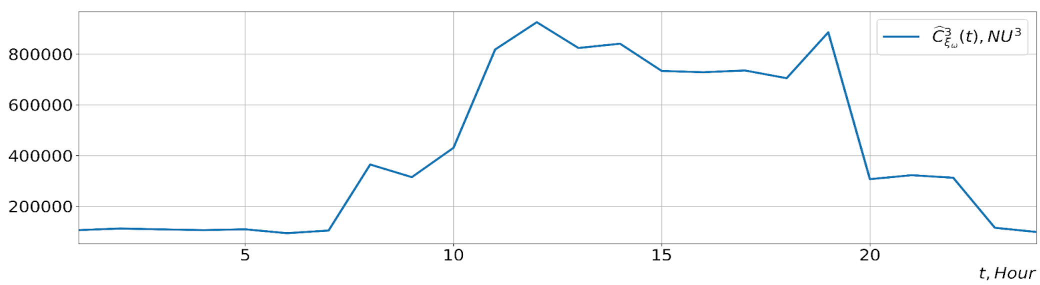

Fig. 6.Graph of realization of statistical estimation of the third-order initial moment function $\hat{C}^3_{\xi_\omega}(t)$.

Thus, the comprehensive analysis of univariate moment functions of the mean, variance, and higher-order moments confirms the inadequacy of stationary models and justifies the necessity of applying the apparatus of cyclicality random processes for modeling and forecasting the electricity consumption process.

The autocovariance function captures the linear dependence between electricity consumption values at different time instances, providing essential information about the temporal correlation structure of the process. The autocovariance function of the proposed electricity consumption signal model is described by the expression:

$$\hat{C}_{p_{\xi_{i_1},\dots,\xi_{i_k}}}(t_1,\dots,t_k)=\frac{1}{M-M_1+1}\cdot\sum_{n=0}^{M-M_1}\left[\xi_{i_{1\omega}}^{R_1}\left(t_1+T(t_1,n)\right)\cdot\ldots\cdot\xi_{i_{k\omega}}^{R_k}\left(t_k+T(t_k,n)\right)\right], i_k=\overline{1,N}.$$

The empirical estimation of the autocorrelation function reveals distinctive patterns that provide insight into the temporal dependencies of the electricity consumption process. The autocorrelation function of the proposed model is described by the expression:

$$\hat{R}_{p_{\xi_{i_1},\dots,\xi_{i_k}}}(t_1,\dots,t_k)=\frac{1}{M-M_1}\cdot\sum_{n=0}^{M-M_1}\left(\xi_{i_{1\omega}}\left(t_1+T(t_1,n)\right)-\hat{m}_{\xi_{i_1}}\left(t_1+T(t_1,n)\right)\right)^{r_1}\cdot\ldots\cdot$$
$$\left(\xi_{i_{k\omega}}\left(t_k+T(t_k,n)\right)-\hat{m}_{\xi_{i_1}}\left(t_k+T(t_k,n)\right)\right)^{r_k},\ i_k=\overline{1,N}$$

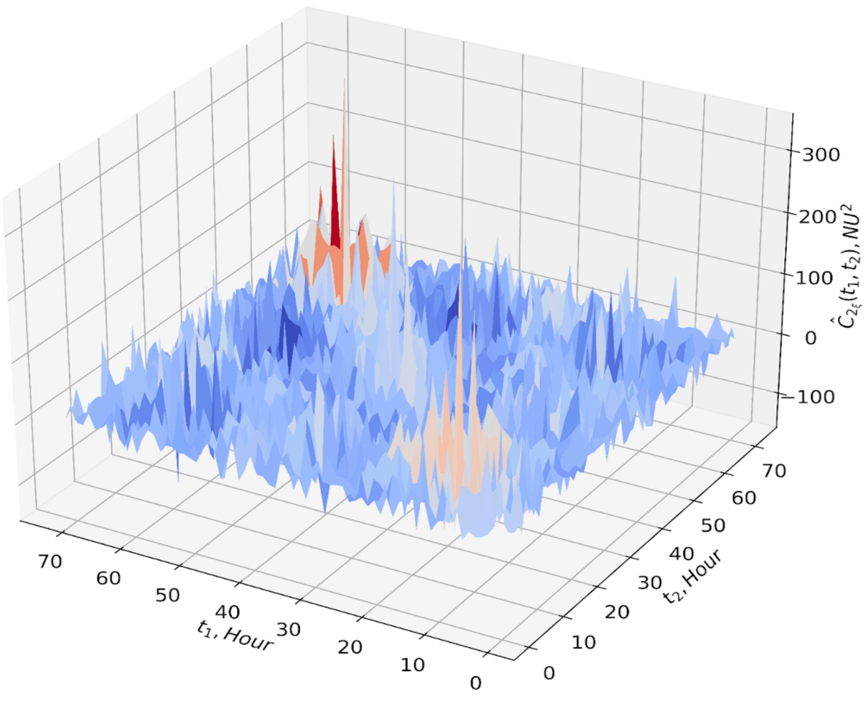

Fig. 6. Graphs of realization of statistical estimates of autocovariance function.

Figure 6 presents an estimate of the autocovariance function $\hat{C}_{2_\xi}(t_1,t_2)$, which determines the linear relationship between electricity consumption values at different moments in time. Its surface has two key features: first, these are peaks along the main diagonal of the plane (t_1,t_2), which correspond to variance and show how variable electricity consumption is at a specific hour of the day; second, and more importantly, this is the presence of repeated off-diagonal peaks. The presence of a peak at specific points means that electricity consumption on a selected day strongly correlates with electricity usage by the residential household on the following day, which can be interpreted as the system's "memory" of its diurnal cycle.

Similar properties are demonstrated by the autocorrelation function $\hat{R}_{2_\xi}(t_1,t_2)$ (Figure 7).

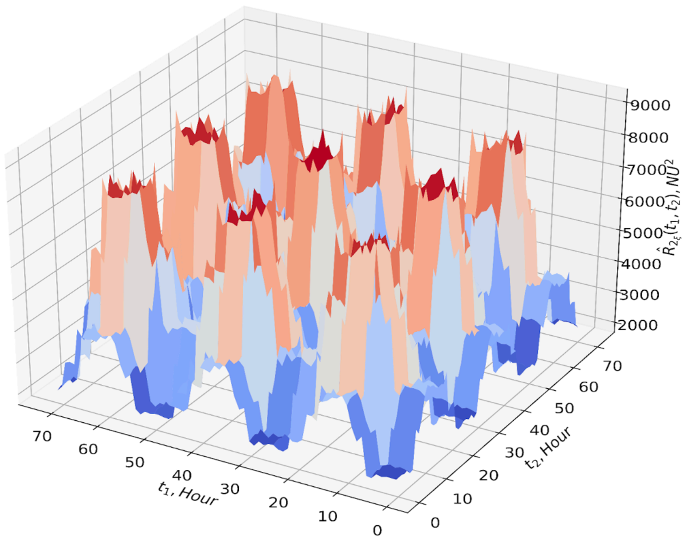

Fig. 7. Graphs of realization of statistical estimates of autocorrelation function.

The surface demonstrates the cyclically repeating structure of the autocovariance function, which is expected since it describes the complete statistical relationship between process values, including the influence of the mean level.

Thus, the comprehensive analysis of second-order correlation functions visually demonstrates that the statistical characteristics of the process change cyclically over time and serves as empirical evidence of its cyclicality nature.

Conclusions. This study addresses the relevant problem of modeling electricity consumption processes to obtain parameters for ensuring stable energy system operation.

We propose representing the electricity consumption process as a cyclicality random process, which enables modeling the stochasticity of the cyclical structure itself, rather than only fluctuations around it. The proposed model, unlike existing approaches, allows structural modeling of the randomness of the cycle parameters themselves - its amplitude and phase.

The conducted empirical analysis of real hourly electricity consumption data confirmed the cyclicality nature of the process. Periodic changes in mathematical expectation, variance, and higher-order moments were revealed, while the specific structure of autocorrelation functions with off-diagonal peaks demonstrates the system's "memory" of the diurnal cycle and confirms theoretical assumptions about the cyclicality nature of the studied process.

The obtained results have important practical significance for modern energy systems, where forecasting accuracy at the level of individual consumers becomes increasingly critical. The proposed approach demonstrates broad application potential for analyzing other cyclical processes across various fields, opening opportunities for interdisciplinary research in the area of stochastic modeling of cyclical systems. Future research will focus on practical implementation of forecasting algorithms based on the proposed model, their validation on diverse datasets, and comparative analysis with existing electricity consumption forecasting methods.

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