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METHOD OF LOCALIZED SIGNAL RECONSTRUCTION IN DYNAMIC ENVIRONMENTS BASED ON MODIFIED VOLTERRA SERIES

Perets K., Zhuchenko O. Method of localized signal reconstruction in dynamic environments based on modified Volterra series. The paper presents a comprehensive study and development of an adaptive method for signal reconstruction in dynamic environments. The proposed method is based on the use of modified Volterra series with temporal constraints, where the contribution of kernels is limited by local time windows defined using a smoothing Gaussian function. This approach overcomes the limitations of traditional spectral methods, which, due to the smoothing effect, are unable to accurately reproduce transient or impulsive features of the signal. To detect critical areas of the signal, an instability indicator is introduced, enabling selective activation of the time-limited model only in unstable zones. In stable regions of the signal, reconstruction is carried out using a frequency model, ensuring efficient use of computational resources. Experimental results show an increase in the local coherence coefficient (ALC) in the range of 10–14%, depending on the spatial localization of critical points and the intensity of temporal signal changes, as well as a decrease in the mean squared error (MSE) by 12–18% compared to traditional frequency-based reconstruction methods. The obtained results confirm the effectiveness of the proposed method for signal processing in cognitive telecommunications systems under complex noise conditions.
Keywords: cognitive telecommunication systems, communication channel, signal, interference, noise immunity, Volterra series, frequency spectrum, reconstruction, optimization, frequency and time domain, Gaussian function.

Перець К. Г., Жученко О. С. Метод локалізованої реконструкції сигналів у динамічному середовищі на основі модифікованих рядів Вольтерра. У статті представлено комплексне дослідження, присвячене розробці адаптивного методу реконструкції сигналів у динамічних середовищах. Запропонований метод базується на використанні модифікованих рядів Вольтерра з часовими обмеженнями, де внесок ядер обмежується локальними часовими вікнами, визначеними за допомогою згладжувальної Гаусової функції. Такий підхід дозволяє подолати обмеження традиційних спектральних методів, які внаслідок згладжувального ефекту не здатні точно відтворювати швидкоплинні або імпульсні особливості сигналу. Для виявлення критичних ділянок сигналу, а саме областей з різкими змінами або локальними аномаліями, в роботі введено індикатор нестабільності, що дозволяє здійснювати вибіркову активацію часово обмеженої моделі лише в нестійких зонах. У стабільних ділянках сигналу реконструкція виконується з використанням частотної моделі, що забезпечує ефективне використання обчислювальних ресурсів. За результатами експериментів отримано зростання коефіцієнта локальної узгодженості (ALC) в діапазоні 10–14% в залежності від просторової локалізації критичних точок та інтенсивності часових змін сигналу, а також зменшення середньоквадратичної похибки (MSE) на 12–18% у порівнянні з традиційними методами частотної реконструкції. Отримані результати підтверджують ефективність запропонованого методу у задачах обробки сигналів для когнітивних телекомунікаційних систем в умовах складного завадового середовища.
Ключові слова: когнітивні телекомунікаційні системи, канал зв'язку, сигнал, завади, завадостійкість, ряди Вольтерри, частотний спектр, реконструкція, оптимізація, частотна та часова область, Гаусова функція.

Statement of a scientific problem.

Traditional signal spectral reconstruction methods based on Volterra series of various orders demonstrate high efficiency in recovering global frequency components, but lose accuracy when processing signals with pronounced local temporal variations.

The smoothing effect of such models leads to the loss of critical fragments containing key information, especially in cognitive radio systems. The lack of adaptive approaches capable of combining frequency-domain reconstruction with localized temporal analysis limits modeling accuracy under dynamically changing conditions.

Therefore, it is relevant to develop an adaptive signal reconstruction method which, unlike classical approaches, enables dynamic activation of a localized model within unstable temporal intervals. This approach preserves essential signal features and enhances reconstruction accuracy in environments with complex and variable structures.

Research analysis.

The review of existing domestic and international studies on modeling nonlinear signal components in the frequency domain has revealed that this topic remains insufficiently explored. Studies [1, 4, 5, 12] have investigated the application of Volterra series for modeling nonlinear systems in the frequency domain, particularly for isolating dominant frequency interactions and improving spectral reconstruction accuracy. However, works [6, 10, 11, 14] have not adequately addressed the limitations of spectral models in reconstructing signals with pronounced local temporal structures, such as impulsive bursts or abrupt amplitude transitions. Publications [2, 3, 7, 9] propose methods of localized time-constrained modeling, including the use of window functions and modified Volterra kernels. Nevertheless, they lack mechanisms for dynamic adaptation to unstable signal segments. In works [8, 13, 15], indicators such as local energy and gradient-based activity are examined for anomaly detection, yet their integration into hybrid time-frequency models remains an open research problem. Therefore, further research is warranted in the areas of automatic critical point detection, integration of local and global models, and reduction of computational complexity for practical use in cognitive radio systems.

The purpose of the work.

The purpose of this study is to develop an adaptive signal reconstruction method based on a localized Volterra series model with dynamic activation, aimed at improving the accuracy of transient feature recovery in nonstationary environments while reducing computational complexity.

Presentation of the main material and substantiation of the obtained research results.

In the article [12], it was substantiated that second-order Volterra series-based signal reconstruction in the frequency domain enables effective modeling of dominant frequency interactions while reducing mean squared error (MSE), even under challenging noise conditions [1,4,5]. However, such spectral models exhibit inherent limitations when applied to signals with pronounced local temporal structures, such as impulsive bursts, narrowband disturbances, or abrupt amplitude transitions. In these scenarios, transformation into the frequency domain often leads to the smoothing of temporal features, which results in the loss of informative components. This limitation is particularly critical in cognitive radio systems, where localized signal variations may carry essential information or trigger immediate system response [6].

To enhance the accuracy of local feature reconstruction in the time domain, the proposed method introduces a constrained Volterra model. Unlike conventional approaches – either linear convolution with a fixed kernel or nonlinear reconstruction using a full Volterra series model – that process the signal across the entire time axis, the proposed framework selectively targets regions characterized by high temporal instability. This strategy not only prevents the smoothing of crucial transient details but also reduces computational complexity and improves reconstruction quality in dynamically varying signal segments [10,11].

In [12], the system output was represented using a discretized *r*-th order Volterra model. However, this formulation did nt incorporate the local nature of signal variability. To address this limitation, we introduce a localized temporal window function $\omega(t, t_0, \Delta)$, which restricts the contribution of the Volterra kernel to a specific neighborhood around a reference time point t_0. The modified localized reconstruction model is expressed as [3,9]:

$$d_{loc}(t_0) = \sum_{r=1}^{R} \sum_{\tau_1=0}^{M} \dots \sum_{\tau_r=0}^{M} h_r(\tau_{1,} \dots \tau_{r,}) \prod_{i=1}^{r} x(t_0 - \tau_i) \cdot \omega(t_0 - \tau_i, t_0, \Delta)\,, \tag{1}$$

here: $x(t)$ – denotes the input signal;

$h_r(\tau_{1,} \dots \tau_r)$ – are the *r*-th order Volterra kernels;

R is the maximum order of the Volterra expansion;

M is the memory length (maximum lag);

τ_i are time-lag indices over which the kernel operates;

$\omega(t, t_0, \Delta)$ – is the binary window function defined as: $\omega(t, t_0, \Delta) = \begin{cases} 1, if\ |t - t_0| \le 0 \\ 0, if\ |t - t_0| > 0 \end{cases}$.

The parameter Δ defines the width of the temporal window centered at t_0, effectively allowing the model to focus only on localized regions of the signal where dynamic features are present. This spatial restriction enhances the sensitivity of the Volterra model to transient behavior while reducing unnecessary computation in stable intervals [3,9]. However, to effectively utilize this localized modeling capability, it is necessary to identify the time points where such transient behavior occurs.

The use of this equation enables the isolation of the temporal context surrounding a suspicious (critical) point, where significant changes in the signal’s derivatives occur that cannot be effectively captured through spectral reconstruction.

To identify time moments t_0, where it is appropriate to activate the local temporal model, the proposed method introduces an analytical instability indicator:

$$K(t) = \left| \frac{d^2x(t)}{dt^2} + \epsilon \frac{dx(t)}{dt} \right|, \epsilon \ll 1. \tag{2}$$

The indicator $K(t)$ combines the signal's curvature and the gradient of local variations, allowing more precise localization of abrupt transitions and anomalies. The coefficient ϵ ensures a balance between sensitivity to discontinuities and robustness against noise. When $K(t) > k$, where k is a detection threshold (experimentally defined according to the expected level of signal variability), the moment $t = t_0$ is identified as a critical point. Such points are characterized by significant changes in the first or second derivative of the signal, indicative of sharp transitions, inflection zones, or local irregularities, that cannot be effectively modeled using global spectral methods. Localized analysis is therefore necessary to preserve the fidelity of transient features in signal reconstruction [2,6].

In practical scenarios, the second derivative of the signal is approximated numerically using a central difference scheme, which offers a computationally efficient and stable method for discrete signal processing. The corresponding finite-difference expression is given by:

$$\frac{d^2x(t)}{dt^2} \approx \frac{x(t+\delta) - 2x(t) + x(t-\delta)}{\delta^2}. \tag{3}$$

This formulation is introduced by the authors specifically for use in localized signal reconstruction scenarios, where analytical differentiation is impractical or noisy. The proposed numerical approach eliminates the need for symbolic operations and is particularly well-suited for real-time applications involving sampled or discretized signals.

It is important to note that the proposed method does not rely on L'Hôpital's rule [16], as the signal derivatives are estimated numerically using finite-difference approximations. This approach eliminates the need for limit-based transitions and is better suited for the analysis of digitally sampled signals.

Furthermore, the proposed framework enables the parallelization of critical point detection and the execution of localized reconstruction exclusively within regions exhibiting pronounced instability. This significantly reduces computational overhead and enhances the model's scalability. As a result, the method exhibits dual adaptivity:

– with respect to the kernel order r, which is selected based on the complexity of the local signal structure;

– and in time, through dynamic activation of the reconstruction process within selected temporal intervals only.

In addition, to improve the accuracy of identifying complex regions within the signal, it is advisable to incorporate its local energy characteristics. For this purpose, the following indicator is computed [13,14].

1. Local energy of the signal $E(t_0)$ within a temporal window centered at point t_0:

$$E(t_0) = \sum_{t=t_0-\Delta}^{t_0+\Delta} x(t)^2. \tag{4}$$

A high value of $E(t_0)$ indicates the presence of an impulse, spike, or transition, whereas a low value corresponds to a stable (slowly varying) segment, where localized reconstruction is generally unnecessary.

2. Variation intensity (gradient-based activity measure):

$$G(t_0) = \sum_{t=t_0-\Delta}^{t_0+\Delta} \left(\frac{dx(t)}{dt}\right)^2. \tag{5}$$

The indicator $G(t_0)$ quantifies the rate of signal variation within the temporal window and enables the detection of rapid oscillations or discontinuities in the signal's derivative [14].

3. Cumulative instability $C(t_0)$ – a generalized indicator of local signal complexity that integrates both energetic and dynamic components:

$$C(t_0) = \Upsilon_1 \cdot E(t_0) + \Upsilon_2 \cdot G(t_0), \qquad \Upsilon_1, \Upsilon_2 > 0, \tag{6}$$

here, Υ_1, Υ_2 – are weighting coefficients that determine the relative contribution of the energy and variation intensity to the overall complexity assessment [10].

Once the coordinates of the critical points t_0, have been identified, localized signal reconstruction is performed within the corresponding temporal windows. However, to avoid abrupt transitions between the localized and global models, the proposed method incorporates a smoothing window kernel based on the Gaussian distribution [9]:

$$\omega(t, t_0, \Delta) = \exp\left(-\frac{(t - t_0)^2}{2\Delta^2}\right). \tag{7}$$

This kernel performs soft weighted filtering, concentrating the majority of the influence around the center of the window at t_0, while gradually attenuating the weights assigned to neighboring time points.

Fig.1 illustrates an example of a Gaussian window with parameter Δ=0,2 applied to a unit impulse signal. As a result, a smoothing effect is observed, where the impulse influences not only the central point but also the adjacent time samples to a lesser extent.

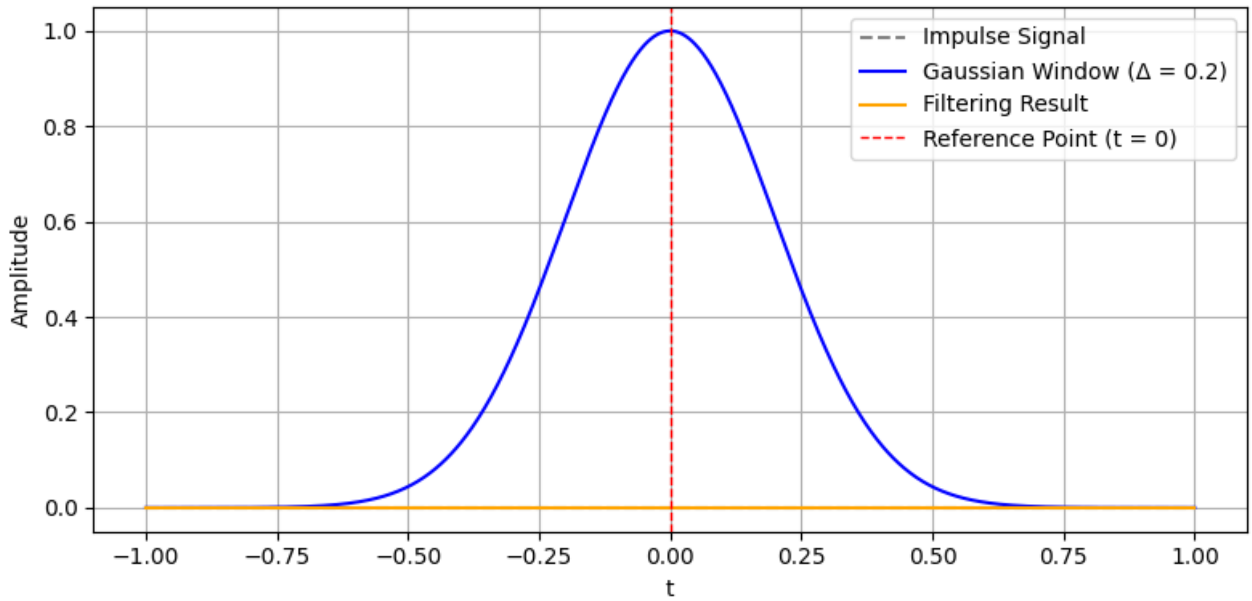

Fig.1 – Impulse filtering using a Gaussian window (Δ = 0,2)

The window spreads the impulse's effect across neighboring points, illustrating the smoothing behavior.

Fig. 2 illustrates how the shape of the Gaussian window changes for different values of the parameter Δ. As *Δ* increases, the window becomes wider, covering a larger time interval while reducing central concentration. Thus, *Δ* serves as a spatial control parameter for the scope of the local model.

To ensure consistent signal reconstruction across both stable and critical regions, the localized model is integrated with the global frequency-based approach. This fusion results in an integrated mathematical reconstruction framework defined as follows:

$$\hat{d}(t) = (1 - \alpha(t)) \cdot d_{freq}(t) + \alpha(t) \cdot d_{loc}(t), \tag{8}$$

where $\alpha(t) \in [0,1]$ is an adaptation function determined by the value of the instability indicator $K(t)$;

$d_{freq}(t)$ is the signal reconstructed in the frequency domain (global model);

$d_{loc}(t)$ is the localized model, activated only near critical points.

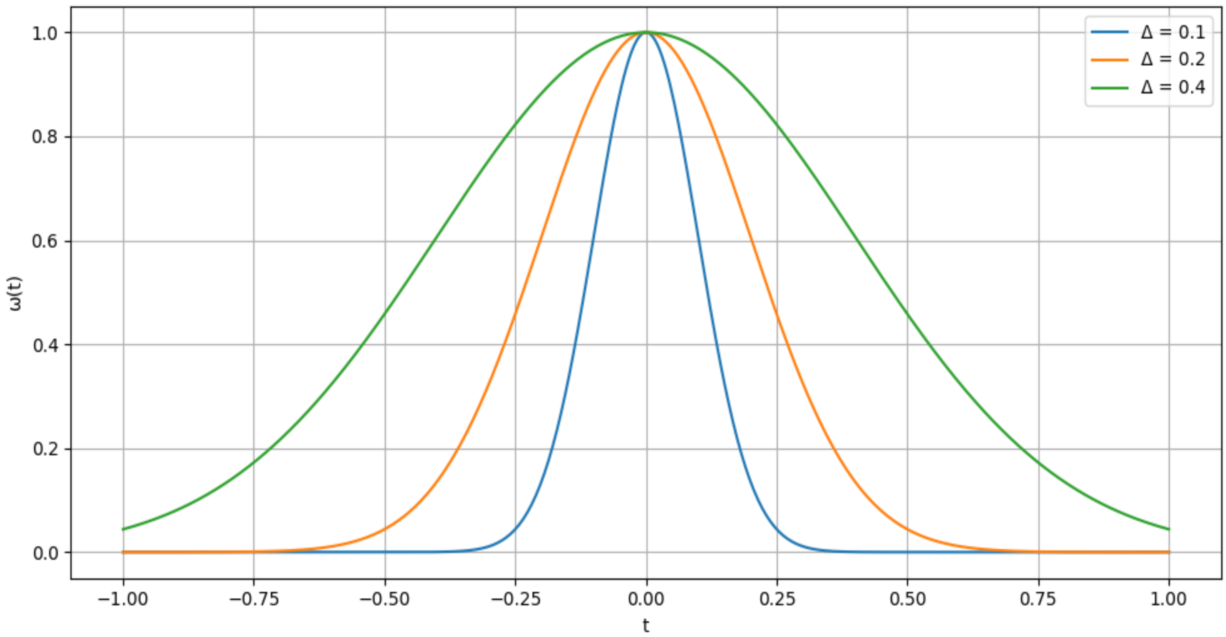

Fig.2 – Comparison of Gaussian windows for different values of Δ

The integration scheme combining frequency-domain and time-domain models using the dynamic weighting coefficient $\alpha(t)$ is proposed by the authors as part of the adaptive reconstruction framework.

The parameter $\alpha(t)$ determines the relative influence of the local and global models on the reconstructed signal:

– when $\alpha(t) \approx 0$, the reconstruction relies primarily on the spectral (global) model;

– when $\alpha(t) \approx 1$, the time-domain, locally adaptive model is dominant.

The value of $\alpha(t)$ changes dynamically depending on the instability indicator $K(t)$, enabling the model to flexibly adapt to signal variations and ensuring a smooth transition between time-domain and frequency-domain reconstruction. This prevents discontinuities or abrupt shifts when switching between regions.

To visualize the principle of adaptive switching between the time-domain and frequency-domain models, Tab. 1 and Fig. 3 present an example of how the parameter *α(t)* changes in response to the instability indicator *K(t)*.

The value of *α(t)* increases in regions with critical signal changes, thereby activating the local model, and decreases in stable segments where global spectral reconstruction is more appropriate.

Table 1 – Adaptation of the signal reconstruction model

t	$K(t)$	$\alpha(t)$	Dominant model	Comment
0,1	0,002	0,05	Frequency-based (global)	Stable signal; frequency model is appropriate
0,35	0,12	0,65	Time-domain (local)	Critical point detected; local reconstruction activated
0,50	0,08	0,40	Mixed mode	Smooth transition between models
0,72	0,15	0,85	Time-domain (local)	Strong instability
0,95	0,005	0,02	Frequency-based (global)	Return to stable regime

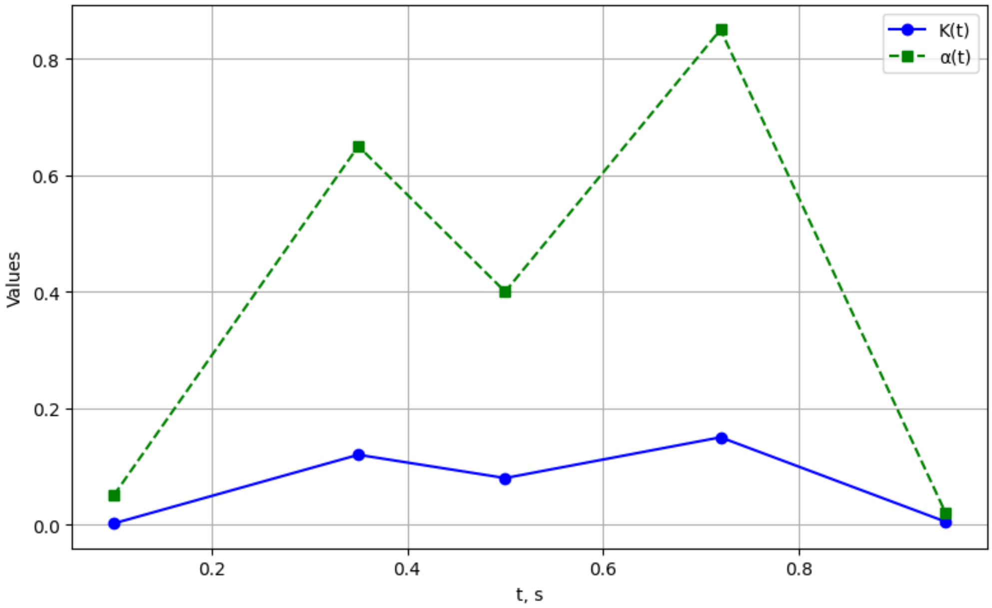

Fig.3 – Dynamics of $K(t)$ and $\alpha(t)$ over time

To validate the effectiveness of the proposed integrated model, it is necessary not only to visualize the dynamics of the adaptation parameters but also to quantitatively assess its impact on signal reconstruction accuracy.

For this purpose, two indicators are calculated: the local alignment coefficient $ALC(t_0)$ and the local mean squared error $MSE_{loc}(t_0)$, which together evaluate the quality of signal restoration near critical points.

$$ALC(t_0) = \frac{\sum_{t=t_0-\Delta}^{t_0+\Delta} x(t) \cdot \hat{d}(t)}{\sqrt{\sum x(t)^2 \cdot \sum \hat{d}(t)^2}}, \quad (9)$$

$$MSE_{loc}(t_0) = \frac{1}{2\Delta} \sum_{t=t_0-\Delta}^{t_0+\Delta} \left(x(t) - \hat{d}(t)\right)^2, \quad (10)$$

Tab. 2 and Fig. 4 present the modeling results at selected critical points of the signal, allowing for a comparative analysis of the adaptive and traditional reconstruction models.

Table 2 – Comparison of signal reconstruction quality

t_0	$ALC(t_0)$ (traditional)	$ALC(t_0)$ (adaptive)	$MSE_{loc}(t_0)$ (traditional)	$MSE_{loc}(t_0)$ (adaptive)
0,20	0,78	0,84	0,022	0,018
0,35	0,76	0,83	0,024	0,020
0,50	0,74	0,81	0,026	0,021
0,72	0,73	0,80	0,028	0,023
0,88	0,77	0,85	0,023	0,018

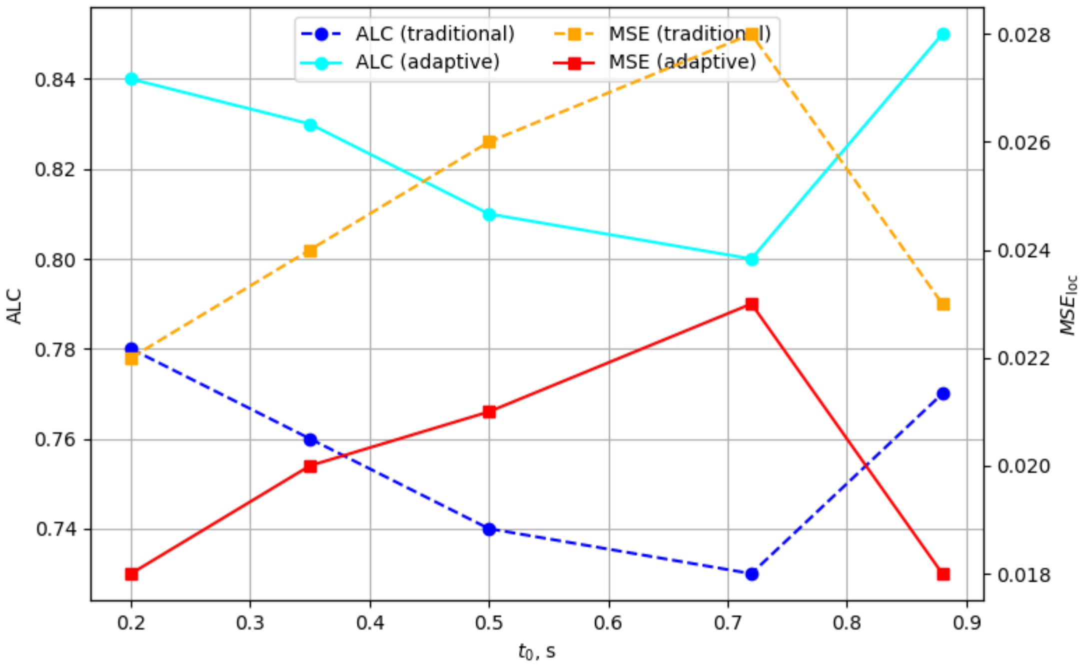

Fig. 4 – Dynamics of $ALC(t_0)$ and $MSE_{loc}(t_0)$ at critical points

As observed from Table 2 and Figure 4, the local alignment coefficient $ALC(t_0)$ increased on average by 10–14%, indicating improved phase and amplitude synchronization of the reconstructed signal. At the same time, the local mean squared error $MSE_{loc}(t_0)$ was reduced by approximately 12–18%, confirming enhanced reconstruction accuracy in dynamically varying segments.

For the purpose of practical implementation, based on the theoretical framework and experimental results presented above, a step-by-step algorithm for adaptive signal reconstruction has been developed (Fig. 5).

Step 1 – Input signal analysis.

The instability indicator $K(t)$ is computed to identify regions with potential abrupt changes in signal structure.

Step 2 – Detection of critical points.

Time instances are selected where the value of $K(t)$ exceeds the predefined threshold k, indicating the need for localized analysis.

Step 3 – Local reconstruction.

At the identified critical points, a localized Volterra model with a Gaussian window is applied to accurately restore signal structure within the corresponding time window.

Step 4 – Global reconstruction.

In stable regions where $K(t)<k$ a frequency-domain model is employed for efficient background signal reconstruction.

Step 5 – Adaptive model fusion.

An integrated signal is formed using the weighting coefficient $\alpha\,(t)$, which dynamically balances the influence of local and global models based on the degree of instability.

Step 6 – Final signal reconstruction.

The resulting signal $\hat{d}(t)$, combines the strengths of both reconstruction strategies, ensuring high accuracy in dynamic segments while preserving smoothness in stable regions.

The proposed method of localized time-domain signal reconstruction, based on a time-constrained modified Volterra series model, demonstrates high effectiveness in restoring critical signal fragments containing impulsive disturbances, narrowband transitions, or local anomalies. A key advantage of the method lies in its adaptivity both in the time domain (via the instability indicator *K(t)* and in structural complexity (through variable kernel order selection). The use of a smoothing Gaussian window enables seamless integration of local and global reconstruction processes, while the weighting coefficient $\alpha\,(t)$ ensures a dynamic response to signal variability.

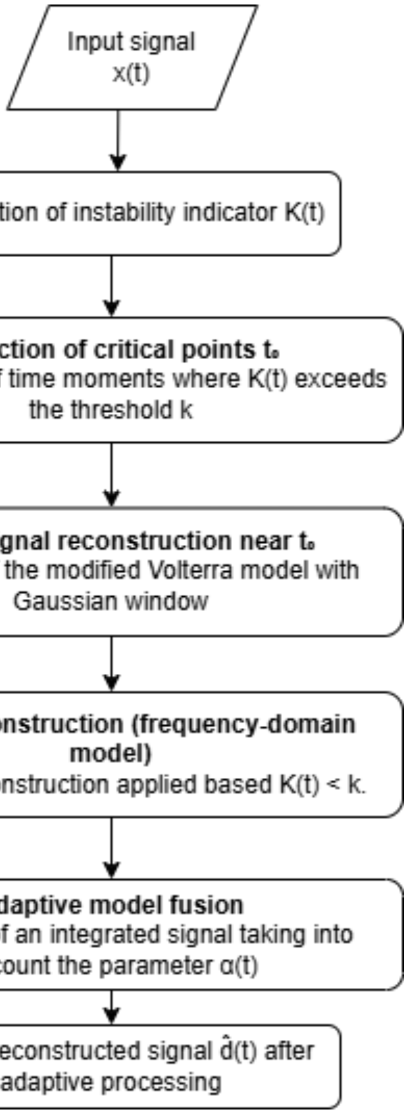

Fig. 5 – Block diagram of the adaptive signal reconstruction algorithm

As a result, the method enhances both phase and amplitude alignment, as reflected by an average increase in the local alignment coefficient $ALC(t_0)$ of 10–14%, and a reduction in the local mean squared error $MSE_{loc}(t_0)$ within the range of 12–18%, which meets the accuracy requirements for dynamic environments. Overall, the method provides structurally consistent signal reconstruction across all time regions, preserving accuracy and information integrity even under noisy or unstable conditions.

Conclusions and prospects for further research.

The study presents a novel adaptive signal reconstruction method that combines localized time-domain modeling with global spectral techniques through dynamic model fusion. The integration of a modified Volterra series constrained by temporal windows and governed by an instability-driven activation mechanism allows for selective processing of critical signal segments. This method contributes to overcoming key limitations of traditional spectral approaches, particularly in contexts where high temporal resolution and localized accuracy are essential.

Future research may focus on optimizing the structure of the adaptive window function, including the automatic tuning of its width Δ based on real-time signal characteristics. Another direction involves generalizing the instability indicator $K(t)$ by incorporating higher-order derivatives or machine-learned features to improve sensitivity and robustness under various noise conditions.

Overall, the presented method establishes a foundation for scalable, data-aware signal reconstruction solutions adaptable to a wide range of practical and high-variability environments.

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