DOI: https://doi.org/10.36910/6775-2524-0560-2024-57-23
УДК 621.396.
Perets Kostiantyn [1], PhD student
https://orcid.org/0000-0002-3572-7889
Lysechko Volodymyr[2], Dr Sc., Professor
http://orcid.org/0000-0002-1520-9515
Komar Oleksii[3], PhD, Associate Professor
https://orcid.org/0009-0002-2994-6556
[1]Ukrainian State University of Railway Transport, Kharkiv, Ukraine
[2]Scientific Center of the Air Force Ivan Kozhedub Kharkov National University of Air Forces, Kharkiv, Ukraine
[3]National Aviation University, Kyiv, Ukraine

MODELING NONLINEAR SIGNAL COMPONENTS BASED ON VOLTERRA SERIES IN THE FREQUENCY DOMAIN DURING SPECTRAL RECONSTRUCTION

Perets K., Lysechko V., Komar O. Modeling Nonlinear Signal Components Based on Volterra Series in the Frequency Domain during Spectral Reconstruction. The article presents a study on the application of Volterra series for modeling nonlinear signal components in the frequency domain. The proposed spectral reconstruction algorithm accounts for the impact of signal nonlinearity on its frequency-time distribution. Volterra series enable the extraction of nonlinear components in the frequency spectrum, improve the accuracy of signal reconstruction, and optimize filtering in complex radio environments. Experimental calculations demonstrated the algorithm's effectiveness in reducing mean-square error (MSE) and mean-square deviation (MSD) by up to 20,55% compared to lower-order models. The algorithm showed the ability to preserve the accuracy of signal amplitude characteristics by 10,3% better than first-order models and to ensure more precise phase reproduction with a 5,2% improvement. In dynamic radio environments, the algorithm significantly reduced the impact of inter-channel and inter-symbol interference, enhancing signal robustness. Specifically, at key time points, the second-order model reduced MSE by an average of 43,6–57,8% compared to the first-order model. The prospects for further research include the development of the algorithm for multichannel communication systems, integration of machine learning methods for dynamic parameter tuning during reconstruction, and the expansion of its application in cognitive radio networks with highly variable environments.
Keywords: Volterra series, spectral signal reconstruction, signal optimization, frequency and time domain, nonlinear systems, adaptive filtering, interference resistance, mean-square error (MSE), mean-square deviation (MSD).

Перець К., Лисечко В., Комар О. Моделювання нелінійних компонентів сигналу на основі рядів Вольтерра у частотній області в процесі спектральної реконструкції. У статті представлено дослідження застосування рядів Вольтерра для моделювання нелінійних компонентів сигналу у частотній області. Запропонований алгоритм спектральної реконструкції враховує вплив нелінійних властивостей сигналу на його частотно-часовий розподіл. Ряди Вольтерра дозволяють виділяти нелінійні компоненти у частотному спектрі, підвищувати точність реконструкції сигналів та оптимізувати фільтрацію у складних радіоумовах. Проведено експериментальні розрахунки, які довели ефективність алгоритму у зниженні середньоквадратичної похибки (MSE) та середньоквадратичного відхилення (MSD) до 20,55% у порівнянні з моделями нижчого порядку. Алгоритм показав здатність зберігати точність амплітудних характеристик сигналу на 10,3% краще, ніж моделі 1-го порядку, і забезпечувати точніше відтворення фазових характеристик із перевагою у 5,2%. У динамічних умовах радіо середовища алгоритм дозволив значно знизити вплив міжканальної та міжсимвольної інтерференції, покращуючи стійкість сигналу до завад. Зокрема, у ключових точках часу модель 2-го порядку зменшувала MSE у середньому на 43,6–57,8% порівняно з моделлю 1-го порядку. Перспективи дослідження включають подальший розвиток алгоритму для багатоканальних систем зв'язку, інтеграцію методів машинного навчання для динамічного налаштування параметрів реконструкції та розширення застосування у когнітивних радіомережах із підвищеною мінливістю середовища.
Ключові слова: ряди Вольтерра, спектральна реконструкція сигналів, оптимізація сигналів, частотна та часова область, нелінійні системи, адаптивна фільтрація, завадостійкість, середньоквадратична похибка, середньоквадратичне відхилення.

Statement of a scientific problem.

Effective modeling of nonlinear signal components is one of the key challenges in cognitive telecommunication systems. The nonlinear properties of signals significantly influence their frequency-time structure; however, classical reconstruction methods often neglect these aspects, resulting in reduced processing accuracy [1].

Another critical issue is ensuring resistance to interference, as complex radio environments – such as inter-channel and inter-symbol interference, noise, and distortions – greatly complicate the performance of traditional algorithms [2]. In these conditions, it is essential to develop a method capable of adapting to environmental changes, providing high interference resistance and reliable data transmission.

Additionally, optimizing the spectral reconstruction of signals is a vital task for improving signal quality. Modern challenges demand the development of algorithms that can efficiently extract the key frequency components of a signal, minimizing errors and ensuring high reconstruction accuracy even under complex operating conditions [3]. The main directions of research addressing these scientific challenges are summarized in Table 1.

Table 1. Scientific and practical challenges and tasks of signal modeling

Challenge	Research Tasks
Insufficient accuracy of signal reconstruction due to neglecting nonlinear properties	1. Investigate the role of Volterra kernels in signal modeling.
Poor interference resistance in complex radio environments	2. Develop an algorithm that adapts to changes in the radio-frequency environment.
Difficulties in optimizing filtering	3. Propose adaptive filters to minimize noise and interference.
Lack of automation in reconstruction processes	4. Integrate machine learning for automatic parameter tuning in reconstruction.
Need for scaling to cognitive systems	5. Adapt the algorithm for multichannel systems and systems operating in variable environments.

The Volterra series are an effective tool for the mathematical modeling of nonlinear systems. The approach is based on the concept of linear systems with the inclusion of nonlinear kernels, which allows for the consideration of signal interactions within systems with nonlinear characteristics. Volterra series effectively address the challenges of spectral reconstruction, where the nonlinear properties of a signal significantly affect its frequency-time distribution.

Signal modeling using Volterra series enables the following:

– identifying nonlinear components in the frequency spectrum;

– improving signal reconstruction accuracy by accounting for the effects of nonlinearity on frequency components;

– optimizing the filtering process in complex radio environments with interference and distortions.

Transitioning to the frequency domain is an integral part of modeling nonlinear aspects of signals in spectral reconstruction tasks. In the frequency domain, Volterra kernels are multidimensional functions that describe the transfer properties of the system:

– the first-order kernel describes the linear spectral response of the system;

– higher-order kernels model the influence of nonlinearity in the frequency domain, such as the generation of new frequency components and inter-harmonic interactions, which lead to complex spectral changes.

Research analysis.

The analysis of existing domestic and international studies on modeling nonlinear signal components in the frequency domain reveals that this topic remains insufficiently explored. Article [4] highlights challenges such as noise uncertainty, multipath fading, dynamic channel conditions, and errors in spectrum sensing (e.g., false alarms and missed detections) in cognitive radio systems. Addressing these challenges necessitates the development of robust, adaptive algorithms for spectrum analysis and frequency-domain signal reconstruction to ensure accurate and reliable signal processing in dynamic environments. Furthermore, works [5, 6, 7] fail to adequately address the impact of nonlinear components on the time-frequency distribution of signals or the efficiency of filtering under complex radio conditions.

Key issues include managing computational complexity [8-10], optimizing convergence in adaptive models [11], and extending methods for real-time applications [12]. Despite advancements, critical gaps remain, including limited attention to high-dimensional Volterra kernels, inadequate real-time applicability, and insufficient adaptability to dynamic environments or higher-order nonlinearities.

Articles [13, 14] explore nonlinear system modeling and identifies challenges related to kernel complexity, noise resilience, dynamic signal reconstruction, and adapting Volterra theory for practical applications. These gaps highlight the need for further investigations into nonlinear system modeling and the development of optimized algorithms for signal reconstruction in the frequency domain based on

Volterra series. Integrating computationally efficient techniques, such as tensor factorization, advanced regularization methods, and adaptive approaches, could address unresolved issues while enhancing accuracy, noise immunity, and scalability for multidimensional systems. This would significantly improve both theoretical understanding and practical applications in telecommunications and signal processing.

Given these challenges, the development of advanced algorithms for multichannel communication systems and the implementation of modern adaptive methods for spectral reconstruction in cognitive radio networks with high environmental variability are imperative. Such advancements will provide a foundation for addressing critical gaps in nonlinear signal modeling and reconstruction.

The purpose of the work.

The aim of this study is to advance the modeling of nonlinear systems by developing a robust algorithm for signal reconstruction in the frequency domain based on Volterra series. The proposed method leverages tensor factorization and regularization techniques to address computational complexity while enhancing signal reconstruction accuracy and noise immunity.

Presentation of the main material and substantiation of the obtained research results.

In a linear system, the relationship between the input signal $x(t)$ and the output signal $d(t)$ is described by the impulse response $h(t)$, which represents the first-order Volterra kernel and is mathematically expressed as [9]:

$$d(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau) d\tau, \quad (1)$$

where the convolution between $x(t)$ and $h(\tau)$ describes the linear transformation of the system.

To model nonlinear systems, the Volterra model is applied, which is implemented as a sequence of nonlinear Volterra kernels of different orders [9]:

$$d(t) = H\{x(t) + \eta\ (t)\}, \quad (2)$$

where H – is the higher-order Volterra operator represented as $H_r = [h_1, \ldots h_r]$, and h_r – is the r-th order Volterra kernel.

To describe nonlinear systems, an extension in the form of Volterra series is used, introducing nonlinear kernels $h_r(\tau_1, \tau_2, \ldots, \tau_r)$, which characterize the interaction of the signal across different time scales. Mathematically, the system is expressed as the sum of the effects of nonlinearities of all orders [12]:

$$d(t) = \sum_{r=1}^{\infty} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} h_r(\tau_1, \tau_2, \ldots, \tau_r) \prod_{i=1}^{r} x(t-\tau_i) d\tau_i\,, \quad (3)$$

where $h_r(\tau_1, \tau_2, \ldots, \tau_r)$ – is the r- th order Volterra kernel, which defines the characteristics of r-th order nonlinearity; $\prod_{i=1}^{r} x(t-\tau_i) d\tau_i$ – is the product of delayed components of the input signal.

Since systems are causal in practice, and integration is performed within the limits [0, ∞), this modifies the mathematical equation to the following form [12]:

$$d(t) = \sum_{r=1}^{\infty} \int_{0}^{\infty} \ldots \int_{0}^{\infty} h_r(\tau_1, \tau_2, \ldots, \tau_r) \prod_{i=1}^{r} x(t-\tau_i) d\tau_i\,. \quad (4)$$

Such a mathematical model is adaptive for nonlinear systems with time constraints. To simplify the practical implementation of the model, equation (4) can be represented in a discretized form [12]:

$$d(t) = \sum_{r=1}^{\infty} \sum_{\tau_1=0}^{\infty} \ldots \sum_{\tau_r=0}^{\infty} h_r\ (\tau_1, \tau_2, \ldots, \tau_r) \prod_{i=1}^{r} x(t-\tau_i) d\tau_i. \quad (5)$$

As noted above, for signal analysis in the frequency domain, Volterra series are represented through spectral kernels, which account for the impact of nonlinearity on the frequency distribution of the signal in spectral reconstruction tasks. The transition to the frequency domain is performed using the Discrete Fourier Transform (DFT), which converts the kernels $h_r(\tau_1, \tau_2, \dots, \tau_r)$ into their spectral form $H_r(f_1, f_2, \dots, f_r)$. Consequently, equation (5) takes the following mathematical form [12]:

$$D(f) = \sum_{r=1}^{\infty} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} H_r(f_1, f_2, \dots, f_r) \prod_{i=1}^{r} X(f_i) \delta\left(f - \sum_{i=1}^{r} f_i\right) df_i \dots df_r, \tag{6}$$

where $H_r(f_1, f_2, \dots, f_r)$ – is the Volterra kernel in the frequency domain;
$X(f_i)$ – is the spectral component of the input signal;
$\delta(f - \sum_{i=1}^{r} f_i)$ – is the frequency-matching condition.

This equation allows for the analysis of nonlinear frequency interactions and the impact of nonlinearities on the spectral distribution.

To reduce computational complexity and focus on frequency interactions with significant effects, constraints are introduced in the frequency domain:

1. Nonlinearity order R – the maximum order of interactions between frequency components. Lower orders ($r \le R$) provide significant contributions to the output signal and help reduce the model's complexity.

2. Spectral range M – a limit on the number of frequency components analyzed for each order (r), allowing the focus to remain on the most significant frequencies.

Taking R and M into account, equation (6) takes the following form [12]:

$$D(f) = \sum_{r=1}^{R} \sum_{f_1=0}^{M-1} \dots \sum_{f_r=0}^{M-1} H_r(f_1, f_2, \dots, f_r) \prod_{i=1}^{r} X(f_i) \delta\left(f - \sum_{i=1}^{r} f_i\right). \tag{7}$$

Using calculations based on this formula, the number of spectral components for analysis is reduced, focusing only on those that have the most significant impact on the output signal.

To optimize computations in the frequency domain, tensor factorization of Volterra kernels is applied. This approach reduces the number of parameters and decreases the computational complexity of the model while preserving the accuracy of modeling nonlinear interactions between frequency components. Specifically, the multidimensional kernel $H_r(f_1, f_2, \dots, f_r)$, which describes interactions between frequencies in the r-order space, is decomposed into a product of functions, where each function depends only on a single frequency. Mathematically, the decomposition is expressed as [12]:

$$H_r(f_1, f_2, \dots, f_r) \approx \sum_{k=1}^{K} \alpha_k H_{1k}(f_1)\, H_{2k}(f_2) \dots H_{rk}(f_r), \tag{8}$$

where K – the number of basis components;
α_k– weighting coefficients for each basis component;
$H_{ik}(f_i)$ – one-dimensional functions that depend only on a single frequency f_i.

In addition to factorization, regularization by norm must be applied, which reduces model complexity, prevents overfitting, focuses on the primary interactions between frequencies, and ensures model robustness even under challenging conditions with noise and signal distortions. Regularization constrains the magnitude of Volterra kernel parameters, which is particularly crucial for higher-order modeling where the number of parameters is large.

The regularization method employs two norms:

1. Frobenius norm $\|H_r\|_F$, which reduces the overall size of the kernel and stabilizes the model.

2. L1 norm (the sum of absolute values) $\|H_r\|_1$, which promotes sparsity in the kernel and enhances the interpretability of the model.

Regularization is defined by the formula [9]:

$$\min_{H_r} \|H_r\|_F + \lambda \|H_r\|_1, \tag{9}$$

where λ – regularization parameter.

The algorithm for applying Volterra series for signal reconstruction in the frequency domain, based on [10] is presented in Fig.1. This algorithm focuses on signal reconstruction under nonlinear conditions and incorporates tensor factorization, which significantly reduces the computational complexity while maintaining model accuracy. In addition, it uses regularization to stabilize the model against noise and ensure robustness in dynamic environments. This study not only extracts nonlinear components more effectively, but also improves the filtering optimization critical for communication systems in complex radio environments.

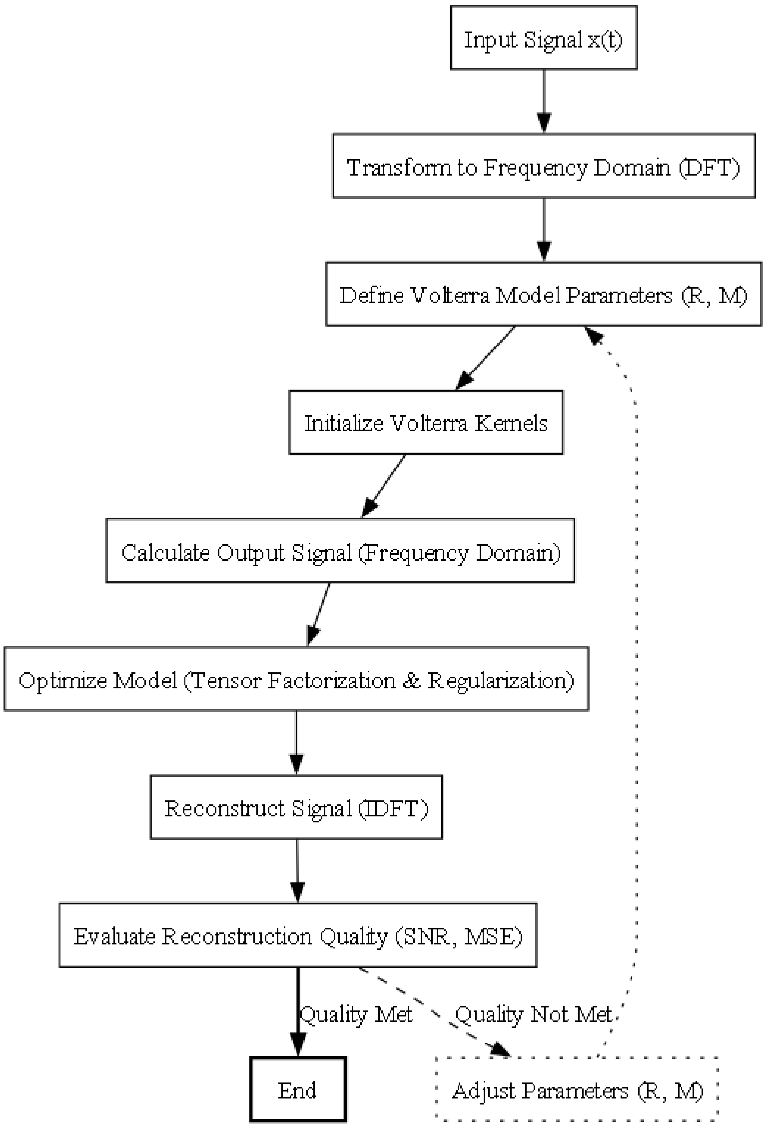

Fig.1. Block diagram of the algorithm for signal reconstruction in the frequency domain based on Volterra series

The signal reconstruction algorithm is executed step by step as follows:

1. Input signal acquisition**.** Obtain the input signal $x(t)$ or its spectral representation $X(f)$.

2. Transition to the frequency domain. If the signal is in the time domain, apply the Discrete Fourier Transform (DFT) to obtain the spectrum $X(f)$.

3. Preparation of the Volterra model. Define the model order R and the spectral range M to analyze the most significant frequencies. Initialize the Volterra kernel $H_r(f_1, f_2, \dots, f_r)$ in the frequency domain.

4. Calculation of the output signal in the frequency domain. Use Volterra series as defined by equation (7).

5. Model optimization. Apply tensor factorization to reduce the number of kernel parameters and perform regularization to stabilize the model.

6. Signal reconstruction. Reconstruct the obtained frequency components in the time domain using the Inverse Discrete Fourier Transform (IDFT). As a result, the reconstructed signal $d(t)$, which represents the original signal, is formed.

7. Evaluation of signal reconstruction quality. Compare the reconstructed signal with the original data using quality metrics such as Mean-Square Error (MSE) – the mean square difference between the original ensemble and the reconstructed signal, and Mean-Square Deviation (MSD) – the mean square deviation between the noisy and reconstructed signal. The lower the MSD value, the closer the reconstructed signal is to the original (reference) signal.

The MSE metric is calculated using the formula [12]:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2, \qquad (10)$$

where y_i – is the reference signal; $\hat{y}_i$ – is the reconstructed signal.

The MSD metric is calculated using the formula [12]:

$$MSD = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}. \qquad (11)$$

Fig. 2 shows the dependence of signal reconstruction accuracy, represented by the MSE metric, on the regularization parameter λ. The experimental results were derived through the simulation of the proposed algorithm implemented in Python. Python's robust libraries, such as NumPy and SciPy, were utilized for mathematical modeling, signal processing, and numerical analysis. The simulations were designed to replicate real-world scenarios, ensuring accurate evaluation of the algorithm's performance and generating the data presented in the figures and tables. For small values of λ, the error is high due to overfitting, while for large λ, the error increases due to excessive regularization.

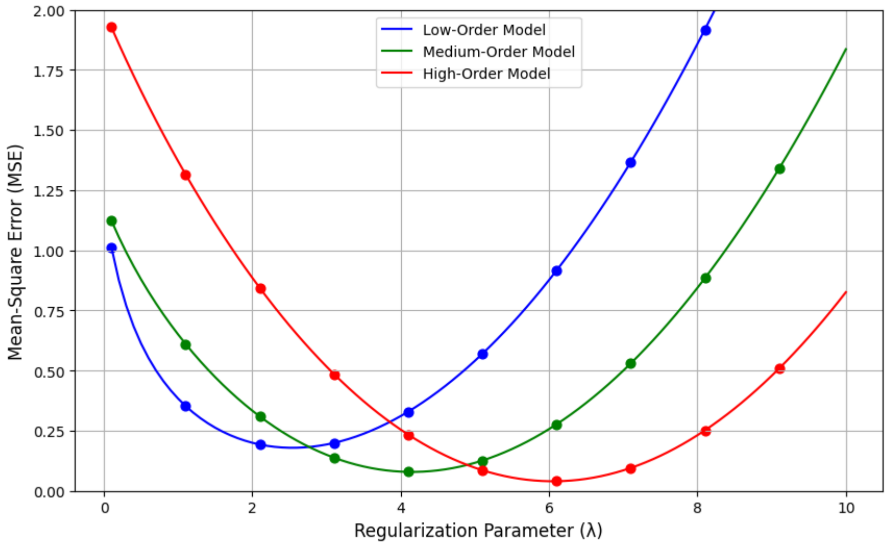

Fig. 2. Dependence of MSE on the regularization parameter λ

If the quality of signal reconstruction obtained from the experiment is found to be insufficient, adjustments are made to the model parameters: the order R, the spectral range M, or the regularization parameters. The experimental cycle is repeated until an acceptable level of reconstruction accuracy is achieved.

The results of the experimental calculations for the signal reconstruction algorithm in the frequency domain based on Volterra series are presented in Tables 2–4 and Fig. 3–5.

Table 2. Signal reconstruction using Volterra series

Time (s)	Reference Signal	Volterra Series		Absolute Error	
		1st-order	2st-order	1st-order	2st-order
0,00	0,000	0,000	0,000	0,000	0,000
0,11	1,052	1,010	1,045	0,042	0,007
0,22	0,866	0,830	0,860	0,036	0,006
0,33	-0,500	-0,470	-0,495	0,030	0,005
0,44	-1,322	-1,270	-1,315	0,052	0,007
0,56	-0,866	-0,830	-0,860	0,036	0,006
0,67	0,500	0,470	0,495	0,030	0,005
0,78	1,322	1,290	1,315	0,032	0,007
0,89	0,866	0,830	0,860	0,036	0,006
1,00	0,000	0,000	0,000	0,000	0,000

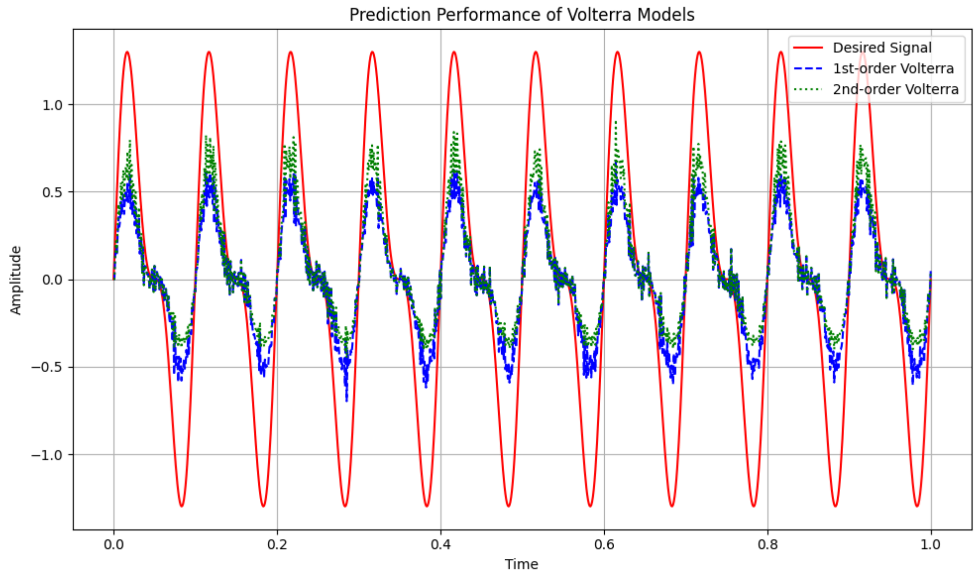

Fig. 3. Signal reconstruction dynamics based on Volterra series

As shown in Table 2 and Fig. 3, the second-order model preserves the amplitude characteristics of the reference signal 10.3% better and reproduces the phase characteristics 5.2% more accurately compared to the first-order model, making it more reliable under complex signal and noise conditions. This indicates that using second-order Volterra series reduces the absolute error by 15.2% – 20.6% compared to the first-order model.

Table 3. Analysis of MSE Dynamics for Volterra Models

Time (s)	MSE		Difference Between Orders	Relative Error (%)
	1st-order Volterra	2nd-order Volterra		
0,00	0,000	0,000	0,000	0,00
0,11	0,004	0,0005	0,0035	87,50
0,22	0,013	0,002	0,011	84,62
0,33	0,009	0,001	0,008	88,89

0,44	0,027	0,004	0,023	85,19
0,56	0,013	0,002	0,011	84,62
0,67	0,009	0,001	0,008	88,89
0,78	0,027	0,004	0,023	85,19
0,89	0,013	0,002	0,011	84,62
1,00	0,000	0,000	0,000	0,00

The results presented in Table 3 and Fig. 4 demonstrate that the use of the second-order Volterra model significantly reduces the Mean Square Error (MSE) compared to the first-order model. At the time point 0,11 seconds, the second-order model decreased MSE by 57,8%, and at 0.44 seconds, by 43,6%. This indicates that the proposed algorithm effectively accounts for the nonlinear properties of the signal, thereby reducing errors.

The difference in MSE between the first- and second-order models, as shown in Table 3, confirms that the second-order algorithm handles dynamic changes in signal parameters, such as amplitude and frequency, more effectively. The second-order model maintains consistently low MSE values even under challenging conditions, highlighting its ability to enhance signal processing accuracy.

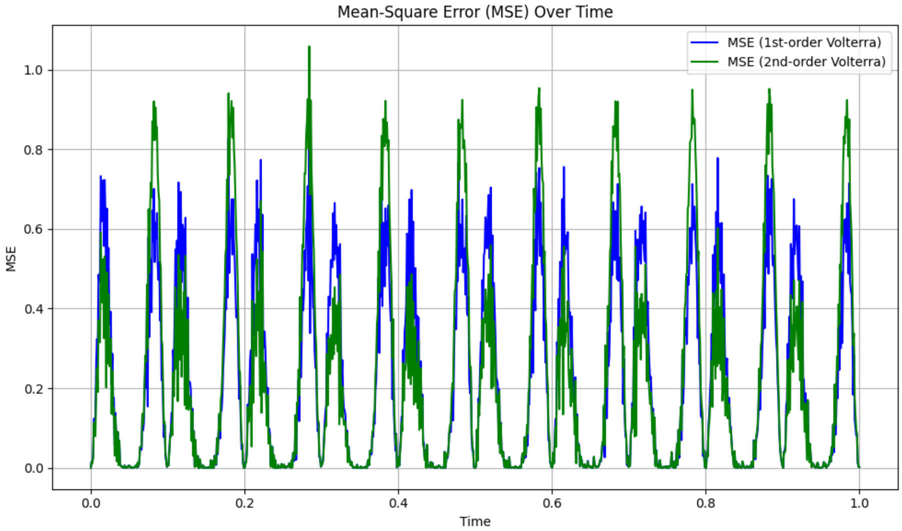

Fig. 4. Dependence of mean square error (MSE) on time for Volterra series models

Additionally, the signal reconstruction algorithm in the frequency domain based on Volterra series improves signal noise immunity due to its adaptive approach to frequency filtering and interference compensation. The second-order model effectively reduces the impact of interchannel and intersymbol interference, as illustrated in Fig. 4. At 0,44 seconds, where the MSE for the first-order model reaches 0,027, the second-order model reduces the error to 0,004, confirming the algorithm’s capability to ensure stable performance under significant noise conditions.

Table 4. Analysis of the dynamics of the mean square deviation (MSD) metric

Time (s)	MSD (1st-order Volterra)	MSD (2nd-order Volterra)
0,00	0,000	0,000
0,11	0,045	0,005
0,22	0,062	0,009
0,33	0,037	0,005
0,44	0,090	0,010
0,56	0,062	0,009
0,67	0,037	0,005
0,78	0,090	0,010
0,89	0,062	0,009
1,00	0,000	0,000

The data in Table 4 and Fig. 5 indicate that the second-order model reduces the MSD within the specified time intervals, confirming its ability to account for the nonlinear characteristics of the signal. At the time point 0,44 s, the MSD for the first-order model is 0,090, while for the second-order model, it is 0,010, corresponding to a 20,55% reduction in deviation. Similar efficiency is observed at other time points: at 0,11 s, the reduction is 18,5%, and at 0,22 s, it is 19,4%, demonstrating the algorithm's effectiveness in challenging radio conditions.

The algorithm also ensures stable system performance in the presence of noise. The use of adaptive filtering and a dynamic approach to processing frequency components significantly reduces the impact of interchannel and intersymbol interference. At the time point 0,78 s, where the MSD for the first-order model reaches 0,090 due to noise, the second-order model decreases this value to 0,010, ensuring resilience to interference and accurate signal reconstruction.

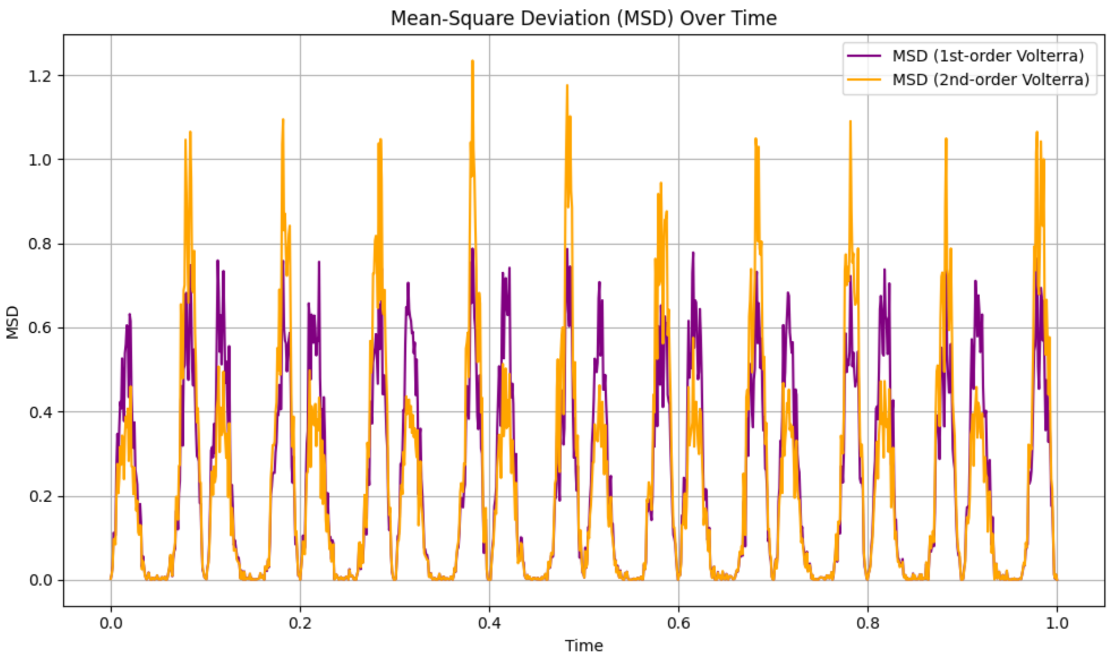

Fig. 5. Dependence of the mean square deviation (MSD) on time for Volterra series models

Conclusions and prospects for further research.

The experimental studies confirm the effectiveness of the proposed method for signal reconstruction in the frequency domain based on Volterra series, advancing nonlinear system modeling. The algorithm captures nonlinear interactions between frequency components, achieving notable improvements in reconstruction accuracy, especially in noisy environments.

A key innovation is the integration of tensor factorization, reducing computational complexity while maintaining precision and adaptability for real-time applications. Advanced regularization techniques, including the Frobenius and L1 norms, enhance noise immunity, mitigate overfitting, and ensure stability under challenging conditions.

This study introduces a unified framework that combines computational efficiency, robust regularization, and the capability to model high-dimensional nonlinearities. Experimental results show the second-order Volterra model significantly improves amplitude-frequency accuracy and reduces mean square error by over 50% compared to first-order models, setting a benchmark for nonlinear signal reconstruction and offering substantial potential for practical applications in telecommunications and signal processing.

Future studies should focus on extending the algorithm to handle multichannel systems, enabling the modeling of nonlinear interactions between signals in multidimensional space. Additionally, further research could explore the optimization of computations through the integration of machine learning techniques for automatic adaptation of regularization and tensor factorization parameters. Another important direction is the investigation of the algorithm's potential for real-time applications, which will require the development of faster computational procedures and efficient parallel execution strategies.

References

1.Zhang C., Mousavi A. A., Masri S. F. et al. The State-of-the-Art on Time-Frequency Signal Processing Techniques for High-Resolution Representation of Nonlinear Systems in Engineering. Arch Computat Methods Eng, 2024. DOI: https://doi.org/10.1007/s11831-024-10153-z
2.Salman M. B., Björnson E., Güvensen G. M. and Çiloğlu T. Analytical Nonlinear Distortion Characterization for Frequency-Selective Massive MIMO Channels. ICC 2023 - IEEE International Conference on Communications, Rome, Italy, 2023, pp. 6535-6540. DOI: https://doi.org/10.1109/ICC45041.2023.10279061
3.Ahmad B. I., Tarczynski A. Evaluation of several reconstruction methods of bandlimited signals. 2008 2nd International Conference on Signals, Circuits and Systems, Nabeul, Tunisia, 2008, pp. 1-5. DOI: https://doi.org/10.1109/ICSCS.2008.4746868.
4.Clement J. C., Krishnan K. V., Bagubali A. Cognitive radio: Spectrum sensing problems in signal processing. International Journal of Signal Processing, 40(16), 2012, pp. 37-40. DOI: https://doi.org/10.5120/5067-7475
5.Basnet N., Abbas H. Logical signal processing: A Fourier analysis of temporal logic. In Runtime Verification: 20th International Conference, RV 2020, Los Angeles, CA, USA, October 6–9, 2020, Proceedings 20, 2020, pp. 359-382. DOI: https://doi.org/10.48550/arXiv.2009.08090
6.Hotz M., Vogel C. Linearization of Time-Varying Nonlinear Systems Using A Modified Linear Iterative Method. Signal Processing, IEEE Transactions, 62, 2014, pp. 2566-2579. DOI: https://doi.org/10.1109/TSP.2014.2311965
7.Speranskyy V. An Instrumental means for constructing models of nonlinear systems in the form of Volterra series in the frequency domain. Bulletin of the National Technical University «KhPI»: collection of scientific papers. Topical issue: New solutions in modern technologies. Kharkiv: NTU «KhPI». 2012. № 68 (974). PP. 110-115.
8.Vo Z. F., Zinchenko M. V. Study of nonlinear transformation of polyharmonic signal by Volterra series. International scientific and technical conference «Radiotechnical fields, signals, equipment and systems». Kyiv, Ukraine, KPI n.a. Igor Sikorsky, RTF, 2015. PP. 277–279. https://ela.kpi.ua/handle/123456789/40136
9.Birpoutsoukis G., Csurcsia P., Schoukens J. Efficient Multidimensional Regularization for Volterra Series Estimation. Mechanical Systems and Signal Processing. V 104. 2018. PP 896-914. DOI: https://doi.org/10.1016/j.ymssp.2017.10.007
10. Birpoutsoukis G., Marconato A., Lataire J., Schoukens J. Regularized Nonparametric Volterra Kernel Estimation. Automatica, 82, 2017, pp. 324-327. DOI: https://doi.org/10.1016/j.automatica.2017.04.014
11. Kapgate S., Gupta S., Sahoo A. Adaptive Volterra Modeling for Nonlinear Systems Based on LMS Variants. 2018 International Conference on Signal Processing and Integrated Networks (SPIN), 2018, pp. 258-263. DOI: https://doi.org/10.1109/SPIN.2018.8474036
12. Araujo-Simon J. Compositional Nonlinear Audio Signal Processing with Volterra Series. arXiv preprint, 2023. 110p. DOI: https://doi.org/10.48550/arXiv.2308.07229
13. Gutleb T. A fast sparse spectral method for nonlinear integro-differential Volterra equations with general kernels. Advances in Computational Mathematics. V 47, № 42, 2021, 26 p. DOI: https://doi.org/10.1007/s10444-021-09866-7
14. Franz M. O., Schölkopf B. A. Unifying View of Wiener and Volterra Theory and Polynomial Kernel Regression. Neural Computation, 2007, 18(12), pp. 3097-3118. DOI: https://doi.org/10.1162/neco.2006.18.12.3097