DOI: https://doi.org/10.36910/6775-2524-0560-2023-52-07
UDC 681.326.74
Krulikovskyi Borys, Phd., Associate Professor
https://orcid.org/0000-0001-7944-551X
Nazaruk Vitalii , к.т.н., Phd., Senior Lecturer
https://orcid.org/0000-0003-3705-5155
Reinska Victoria, Phd., Associate Professor
https://orcid.org/0000-0002-3969-2054
National University of Water and Environmental Engineering, Rivne, Ukraine

DEVELOPMENT OF THE MATHEMATICAL APPARATUS OF BOOLEAN DERIVATIVES TO IMPROVE THE RELIABILITY AND VALIDITY OF INFORMATION PROCESSING.

Krulikovskyi B., Nazaruk V., Reinska V. Development of the mathematical apparatus of Boolean derivatives to improve the reliability and validity of information processing. The article substantiates the concept of the Boolean derivative of the generalized logic model (GLM) with respect to the fault parameter and the basis of its use for solving technical diagnostics problems. Since such a GLM describes the occurrence of faults of arbitrary multiplicity, the diagnostic tests built using this model allow quick identification of the technical condition of the logic circuit, and therefore reduce the recovery time and improve the reliability and validity of information processing.
Key words: logic circuits, technical diagnostic, Boolean derivatives, reliability, validity, information processing.

Круліковський Б.Б., Назарук В.Д., Рейнська В.Б. Розвиток математичного апарату Булевих похідних для покращення захищеності та надійності обробки інформації. В статті обгрунтоване поняття булевої похідної узагальненої логічної моделі (УЛМ) відносно параметра несправності та основи її використання для розв'язку задач технічної діагностики. Оскільки така УЛМ описує появу несправностей довільної кратності, то діагностичні тести, що побудовані з використанням вказаної моделі, дозволяють швидко ідентифікувати технічний стан логічної схеми, а значить скоротити час відновлення і підвищити надійність та достовірність обробки інформації.
Ключові слова: логічні схеми, технічна діагностика, булеві похідні, надійність, валідність, обробка інформації

Formulation of a scientific problem. Improvements based on the microminiaturization of computer equipment lead to an increase in the packing density of elementary logic gates on the crystal and, as a result, to an increase in the complexity of the logic structure of the microcircuit. Increasing the complexity of logic circuits requires a significant increase in computing power to solve the issues of their technical diagnostics, which will certainly arise at the stage of operation of such devices as part of computer systems. In turn, the formalization of methods for building algorithms for diagnosing the technical condition of any object requires the presence of a mathematical model of a computing device, the processing of which makes it possible to solve the entire list of direct and inverse problems of their technical diagnosis.

Such a formal description is usually called a mathematical model of the diagnostic object [1]. On the basis of such mathematical models, direct and inverse problems of technical diagnostics are formulated and solved, which are the basis of procedures for ensuring sufficient reliability parameters of technical means during their intended operation.

It is the effective solution of direct tasks of technical diagnostics of serviced hardware that can significantly reduce the recovery time of a digital device, and thus increase the reliability of information processing, i.e. its security in the face of accidental or specially organized defects by intruders.

When the degree of integration of the elemental basis of discrete computing equipment increases, the probability of occurrence of not only single but also multiple malfunctions in the production, storage and operation of such systems increases. Therefore, modern approaches to solving problems of technical diagnostics cannot be limited to predicting the occurrence of single malfunctions or malfunctions of a certain multiplicity. The closest to reality can be taking into account the uncertain technical condition of the diagnostic object, in which the presence of an arbitrary number of constant malfunctions is possible [2].

Constant malfunctions are understood as malfunctions that lead to the fixation of signal values in arbitrary places of the logic circuit of the digital device by constants 0 or 1.

For modeling such systems, generalized logic models (GLM) of digital circuits [3, 4] were widespread, in which the possible appearance of constant defects is described by changing the values of fault parameters, have become sufficiently widespread.

Particularly attractive is the use of an implicit mathematical model in the form of a structural system of Boolean functions, which stores information about the relationships between the nodes of the device's logic circuit. This makes it possible to immediately take into account the information about the location of the defect when building a verification test, bypassing the process of synthesizing a diagnostic test. This significantly reduces the cost of implementing the process of technical diagnostics of the device under test. This reduces the time required to restore the object under test, thus increasing its reliability and the degree of protection of the information it processes.

The development of diagnostic support for complex computer systems requires solving two types of problems. The development of diagnostic support for complex computer systems requires solving two types of problems. The first group consists of direct tasks of technical diagnostics, which consist in determining the actual technical condition of the object of diagnosis by conducting diagnostic experiments on it. The problem is solved by applying test signals to the inputs of the device and measuring the system's reactions to these signals. by him. The problem is solved by applying test signals to the inputs of the device and measuring the system's reactions to these signals. Analysis of reactions to test signals allows to determine the actual technical condition of the system. The second class of technical diagnostics tasks are inverse tasks, thet consisting in the calculation of test signals and procedures for carrying out the specified diagnostic experiments.

To solve inverse problems of technical diagnostics, the concept of Boolean difference [5] with respect to a logical variable in the expression of a Boolean function performed by a digital circuit is often used. The Boolean derivative expression is set to 1 Boolean derivative of a logical variable

$$\frac{df}{dx_i} = f(x_1, x_2, \dots, x_{i-1}, 1, x_{i+1}, \dots, x_n) \oplus (x_1, x_2, \dots, x_{i-1}, 0, x_{i+1}, \dots, x_n) = 1 \quad (1)$$

defines the conditions under which the function f necessarily changes its value when the value of the variable x_i changes.

That is, equation (1) is a condition for significant dependence of the function f on the value x_i. The prospect of using a powerful Boolean derivatives apparatus for analyzing mathematical models in order to obtain maximum information about the technical state of a digital system is attractive.

In the reviewed works, there is no theoretical justification of the possibilities of using the mathematical apparatus of Boolean derivatives to construct verification tests for checking the serviceability of logical circuits that make up the basis of digital systems. Therefore, the purpose of this work is to substantiate the mathematical apparatus and develop the principles of calculating the conditions for detecting constant faults of arbitrary multiplicity in logic circuits of the Boolean basis. The proposed mathematical apparatus should be used to improve the security and reliability of digital systems of an uncertain technical condition.

Requirements for mathematical models of digital devices. Implicit mathematical models of diagnostic objects [1] are often used to solve problems of technical diagnostics, among which . In order to reduce the volume of calculations, the following requirements for the description of the investigated circuit of the logical device are natural:

1) ease of building a model based on the structural system of logical functions that describe the operation of a good logical circuit;
2) complete consideration of all possible constant irregularities;
3) minimal complexity of the mathematical model;
4) algorithmic simplicity of building models describing the operation of a working logic circuit as well as a circuit with malfunctions of arbitrary multiplicity;
5) absence of redundant parameters of the mathematical model;
6) suitability for computer processing.

Meeting such requirements ensures the maximum efficiency of mathematical model processing when solving technical diagnostics problems [6].

The process and example of building a mathematical model of this type is given in [7]. The mathematical model of the combination adder presented in [7] contains a system of generalized logical models of the structural system of Boolean functions, the output signals of which depend not only on the input x_i and internal y_j functional signals of the circuit, but also on the formal parameters α_l and β_k, which are intended for modeling constant faults on the corresponding buses of the scheme.

The used modeling method involves specifying for each line of the circuit the values of two fault parameters (PN), as given in system (2):

$$\tilde{x_i} = \alpha_l \cdot x_i \vee \beta_k = \begin{Bmatrix} x,\ good\ technical\ condition\!: \alpha = 1, \beta = 0; \\ 1,\ defect\ \equiv 1\!: \quad \alpha = 1, \beta = 1; \\ 0,\ defect\ \equiv 0\!: \quad \alpha = 0, \beta = 0; \end{Bmatrix}. \quad (2)$$

With this approach, the appearance of a single defect is simulated by a change in the value of one fault parameter compared to the good technical condition of the circuit. In order to simulate a double defect, it is necessary to change two parameters of the malfunction in the model of a working technical condition. In this way, the above requirements for the model number 4 and 6 are satisfied.

It is advisable to use this property of the proposed mathematical model to solve the inverse problem of technical diagnostics based on the following theoretical provisions.

Introduction to the mathematical apparatus of Boolean derivatives

The first-order Boolean derivative $\frac{df}{dx_i}$ of the Boolean function f on the variable $\mathbf{x_i}$ is calculated as follows [5]

$$\frac{df}{dx_i} = f\ (x_1, \dots, x_{i-1}, 1, x_{i+1}, \dots, x_n) \oplus f(x_1, \dots, x_{i-1}, 0, x_{i+1}, \dots, x_n)$$

where: **f** $(x_1,x_2,\dots,x_{i-1},1,x_{i+1},\dots,x_n)$ is a unit residual function obtained by substituting the constant 1 for $\mathbf{x_i}$,

-$f(x_1, \dots, x_{i-1}, 0, x_{i+1}, \dots, x_n)$ is a zero residual function obtained by substituting the constant 0 for $\mathbf{x_i}$.

The Boolean derivative of x_i = 0 if f does not depend on x, the Boolean derivative of x_i = 1 if **f** depends only on x, i.e., the first-order Boolean derivative determines the conditions under which the function changes its value when the value of the variable x changes.

In other words, the expression of the Boolean derivative of a logical function by one of its arguments determines the conditions for the significant dependence of the specified function on this argument.

To analyze the significant dependence of the generalized vortex function on the fault parameters, it is proposed to calculate the Boolean derivative of the generalized output function by the fault parameter that models the appearance of a constant defect. To do this, the following concept of the Boolean derivative of the generalized output function by fault parameter can be introduced.

The concept of the Boolean derivative of a generalized output function.

For further presentation, it is advisable to introduce the following notation. All input variables of a logical function are denoted by the symbol of the generalized variable $æ_l\ (l = \overline{1,L})$, where $L\ = n + |A| + |B|$ is the total number of generalized variables equal to the sum of the number of **n** function arguments, the number |A| of parameters of type α_j, the number |B| of parameters of type β_p. The set of fault parameters should be called the technical condition vector, or TC vector, the coordinates of which are all the parameters of the circuit fault:

$$E = \left(\alpha_1, \alpha_2, \dots, \alpha_{|A|},\ \beta_1,\ \beta_2, \dots,\ \beta_{|B|}\right). \tag{3}$$

The symbol E denotes a set of parameters on which the technical state of a logic circuit depends. In this case, the generalized variable æl can be a real function variable, or it can be a fault parameter. With such notations, the generalized output function of a logic circuit is a dependency:

f$(x_1, x_2, \dots, x_n, e_1, \dots, e_2, \dots, e_{|E|})$.

Definition 1: The Boolean derivative of a generalized output function is the expression

$$\frac{d\tilde{f}}{dæ_i} = \tilde{f}\left(æ_1, \dots, æ_{i-1}, 1, æ_{i+1}, \dots, æ_{|L|}\right) \oplus \tilde{f}\left(æ_1, \dots, æ_{i-1}, 0, æ_{i+1}, \dots, æ_{|L|}\right). \tag{4}$$

The Boolean derivative of the generalized output function with respect to the parameter $æ_i$ is 0 if the generalized function $\tilde{f}$ does not depend on $æ_i$. The Boolean derivative with respect to $æ_i$ is 1 if the function depends only on $æ_i$. Let the logical element conjunctor perform the function $z =\ x_1 \cdot x_2$. Taking into account possible constant faults, it is described by a generalized output function:

$$\tilde{z} =\ (x_1 \vee \beta_1) \cdot (x_2 \vee \beta_2) \cdot \alpha_1 \vee \beta_3. \tag{5}$$

or in terms of generalized variables:

$$\tilde{z} =\ (x_1 \vee æ_1) \cdot (x_2 \vee æ_2) \cdot æ_3 \vee æ_4.$$

The following notations are used here: $æ_1 = \beta_1$, $æ_2 = \beta_2$, $æ_3 = \alpha_1$, $æ_4 = \beta_3$.

According to definition (2), the equality of 1 of the Boolean derivative of a logical function with respect to any variable is a necessary and sufficient condition for the specified function to depend significantly on this variable. This means that a change in the value of the parameter $æ_i$ will necessarily cause a change in the value of the function. If x_i is an argument of the function **z**, then the Boolean difference ${dz}/{dx_i}$ determines the significant dependence of the function **z** on the value of the argument **x**. If the

Boolean differential is calculated by the fault parameter $æ_i$, then the Boolean derivative determines the significance of the defect modeled by this parameter for the original function **z**. Thus, for $\tilde{z}$ to depend significantly on $æ_1 = \beta_1$, it is necessary and sufficient to fulfill the condition:

$$\frac{d\tilde{z}}{æ_1} = 1.$$

Taking into account (4), we obtain the following expression of the Boolean derivative of the generalized function $\tilde{z}$ with respect to the generalized variable (fault parameter) β_1 :

$$\frac{d\tilde{z}}{\beta_1} = \overline{x_1} \cdot \alpha_1 \cdot \overline{\beta_3}(x_2 \vee \beta_2). \qquad (6)$$

Let us determine under what conditions the parameter β_1 significantly affects the function $\tilde{z}$. In order for the generalized function $\tilde{z}$ to significantly depend on the fault parameter β_1, it is necessary to satisfy the following requirement:

$$\frac{d\tilde{z}}{\beta_1} = 1$$

The resulting expression of the Boolean derivative of the generalized function $\tilde{z}$ in terms of the fault parameter β_1 can take the value 1 if the following conditions are met, as determined from (5):

1) the zero signal $x_1=0$ must be applied to the input of the connector;
2) parameter $\alpha_1=1$, i.e., the circuit has no ≡0 defect at the valve output;
3) parameter $\beta_3 =0$, i.e. the circuit has no defect ≡1 at the valve output.
4) either set the signal =1 at input x_2, or artificially create a constant defect in the circuit, the model of which is $\beta_2 = 1$.

This means that a defect x_1≡1 in the parameter β_1 distorts the output function z and can be identified (detected) when the functional conditions are met (the set $x_1 = 0$, $x_2 = 1$is supplied to the inputs of the connector), where the output signal of the faulty element is 0. The second group of fault conditions describes the technical condition of the connector circuit, when the defect x_1≡1 in the parameter β_1 is manifested at the output. This state is described by the following values of the fault parameters: $\alpha_1=1$, $\beta_3=0$, i.e., there are no defects in the circuit for parameters α_1 and β_3. This defines the technical conditions for the fault to occur.

Thus, the Boolean derivative of the generalized output function of a combination circuit by a variable or by a fault parameter formulates two types of conditions for the sensitivity of the circuit to changes in the value of the output signal: functional conditions (FC) define the input binary signals required for the defect to occur, and technical conditions (TC) define the technical state of the circuit under which the specified sensitivity is maintained and the fault signal is transported to the output of the circuit, where it can be observed.

In order to use these properties of generalized output functions to solve technical diagnostics problems, it is advisable to formulate several definitions and prove several statements that will allow more efficient solution of technical diagnostics problems of digital devices.

Boolean derivatives of generalized logic models.

Let the generalized logic model (GLM) of the Boolean function F(X, E) [7] be given, where: X = (x_1, x_2, ..., x_n)- n-dimensional vector of input logical variables (arguments) of the function F;

E = (e_1, e_2,..., $e_{|E|}$) is a vector of the technical state of the circuit containing $|A|$ fault parameters α_j

(j=$\overline{1,|A|}$) and |B| fault parameters β_q (q=$\overline{1,|B|}$),

$|E| = |A| + |B|$.

Definition 2. The Boolean derivative of a generalized function $\tilde{F}$ with respect to the parameter e_k is an expression:

$$\frac{d\tilde{F}}{de_k} = \tilde{F}(X, e_1, \dots, e_k, \dots, e_{|E|})\oplus\tilde{F}(X, e_1, \dots, \overline{e_k}, \dots, e_{|E|}). \qquad (7)$$

Since all e_k are Boolean parameters, and $\tilde{F}$ is a Boolean function, all the properties of the Boolean difference dF/dx_i are inherent in $d\tilde{F}/d\,e_k$ .The most important property of the Boolean derivative of the fault parameter is its inequality 0 if the expressions of the function $\tilde{F}$ do not coincide at e=0 and e=1. If these expressions do coincide, then $\frac{d\tilde{F}}{de_k} = 0$, which means that $\tilde{F}$ is independent of e_k.

Definition 3. The function $\tilde{F}(X, E)$ does not depend on the value of the parameter ek if $\tilde{F}$ does not change when the value of e_k is reversed, i.e. if

$$\tilde{F}(X,\ e_1, \dots, e_k, \dots, e_{|E|}) = \tilde{F}\left(X, e_1, \dots, \overline{e_k}, \dots, e_{|E|}\right).$$

Definition 2 can be expressed by the following theorem.

Theorem 1. For the ULM to be independent of the technical state of a digital device determined by the fault parameter e_k, it is necessary and sufficient to fulfill the following condition

$$\frac{d\tilde{F}}{de_k} = 0.$$

The proof of the theorem is based on the property $\tilde{F} \oplus \tilde{F} = 0$ and Definition 2. The dependence of $\tilde{F}$ on the technical state of the logic circuit is described by the condition:

$$\frac{d\tilde{F}}{de_k} \neq 0.$$

In this case, we obtain the following consequences of Theorem 1.

Corollary 1. If.

$$\frac{d\tilde{F}}{de_k} = 0,$$

then the defect modeled by the parameter e_k is insignificant for the output function F.

Corollary 2. If.

$$\frac{d\tilde{F}}{de_k} = 1,$$

then the defect modeled by the parameter e_k is always significant for the output function F, and thus always detected at any values of the input vector X and the TS vector E.

Corollary 3. If.

$$\frac{d\tilde{F}}{de_k} = G(X), \quad (8)$$

then the defect modeled by the parameter e_k is significant for the output function F for any X_h for which $G(X_h) = 1$

Corollary 4. If.

$$\frac{d\tilde{F}}{de_k} = G(E), \quad (9)$$

then the defect modeled by the parameter e_k is significant for the output function F only at those values of the TS vector Em for which $G(E_m) = 1$ regardless of the values of the input vector X.

Corollary 5. If.

$$\frac{d\tilde{F}}{de_k} = G(X, E), \quad (10)$$

then the fault modeled by the parameter e_k is significant for the output function F only in the technical states described by the TS vector E_m , and at such values of the input vector X_k for which $G(X_k, E_m) = 1$.

Thus, in general, to determine the significance of a certain variable or fault parameter for the generalized output function $\tilde{F}$, it is necessary to specify the vectors X and E. The vector X determines the values of the input variables x_i , and the vector E determines the values of the fault parameters (technical states of the circuit) at which the parameter e_k significantly affects the operation of the circuit.

Definition 4. The set of values X_k of the input variables at which the Boolean derivative of the generalized output function by e_k takes a single value is called the functional conditions (FC) of the significance of the parameter e_k for the output function.

Definition 5. The set of values E_m of the TS vector at which the Boolean derivative of the generalized output function by e_k takes a single value is called the technical conditions (TC) of the significance of the parameter e_k for the output function.

Summarizing the above, the following can be stated. The essentiality of the generalized variable $æ_l$ for the generalized output function $\tilde{F}$ of the logic circuit is ensured by fulfilling the functional and technical conditions. These conditions are determined from the equality 1 of the Boolean derivative of the generalized function $\tilde{F}$ of the circuit in terms of the generalized variable $æ_i$

$$\frac{d\,\tilde{F}}{dæ} = 1$$

If we take the fault parameter as a generalized variable $æ_i$, then equality 1 of expression (3) defines the conditions under which the defect modeled by the generalized parameter $æ_i$ distorts the output signal of the circuit.

Conclusions.

1. In the presented work, an extended concept of the Boolean derivative is proposed, namely, the concept of the Boolean derivative of the generalized output function of a digital combinational device is substantiated and formulated, which is represented by an optimized analytical description in the form of a generalized logic model (GLM). The used GMM is optimized in terms of complexity, redundancy, and algorithmic processing by computing tools [3].

2. It is proposed to use the mathematical apparatus of Boolean derivatives with respect to the generalized arguments of the GLM as a mathematical apparatus for calculating the significance of the arguments of the logical function being performed

$$\frac{d\,\tilde{F}}{dæ}$$

3. As such arguments, the input signals of the logic circuit x_i that implements the function or the formal fault parameters e_k, which are introduced into the FEM and are intended to model constant circuit defects, can be used.

4. The definitions, theoretical prerequisites, analytical expressions, and the validity of the formulated statements about the effectiveness of using Boolean derivatives for the argument of the function x_i and the fault parameter e_k are given.

5. All variants of determining the conditions for the dependence of the GLM on the generalized parameters that determine the law of its functioning in the event of possible occurrence of constant-type defects are considered.

6. The proposed mathematical apparatus can be effectively used to solve inverse problems of technical diagnostics of digital devices described by logic circuits.

7. The conditions for checking the serviceability of a logic circuit determined by the proposed apparatus contain functional and technical conditions for detecting constant defects regardless of the multiplicity of faults present in the circuit, which allows building verification and diagnostic tests of digital information processing devices.

8. The use of the proposed apparatus makes it possible to increase the security of information processing in digital devices by increasing reliability by reducing the time of hardware recovery in the event of constant faults.

References

1. Fundamentals of technical diagnostics / V.V. Karibsky, P.P. Parkhomenko, E.S. Sogomonyan, V.F. Khalchev - Moscow: Energia, 1976. 464 p.
2. Kaznacheev V.I. Model of a digital automaton with faults. Izv. of the USSR Academy of Sciences. Ser. Techn. cybernetics, 1968, No. 6, pp. 55 - 62.
3. Krasnov, V.V.; Krulikovskiy, B.B. Mathematical modeling of combinational circuits with regard to possible faults. Hybrid computers and complexes. Issue 3. - Kyiv: Scientific thought. - 1980. - p.26 - 32.
4. Sklyarevich A.N., Sklyarevich F.A. Linear systems with possible changes - Riga: Zinatne, 1985.
5. Bessonov A.A. et al. Automation of construction of control tests. Edited by A.A. Bessonov. Л.: Energia, 1976. -224 c.
6. Krulikovsky B.B. Estimation of the complexity of generalized mathematical models of discrete devices. Integrated intelligent robotic systems (IIRTC-2011). Fourth International Scientific and Practical Conference, May 23-25, 2011, Kyiv, Ukraine (collection of abstracts) - K.: NAU, 2011. - pp. 316-318.
7. Krulikovskyi B.B. / Mathematical modeling of the combinational adder in the tasks of technical diagnostics. Krulikovsky B.B., Reinskaya V.B. // Computer-integrated technologies: education, science, production. № 48. Lutsk, LNTU. - C.135-140. - 2022p.